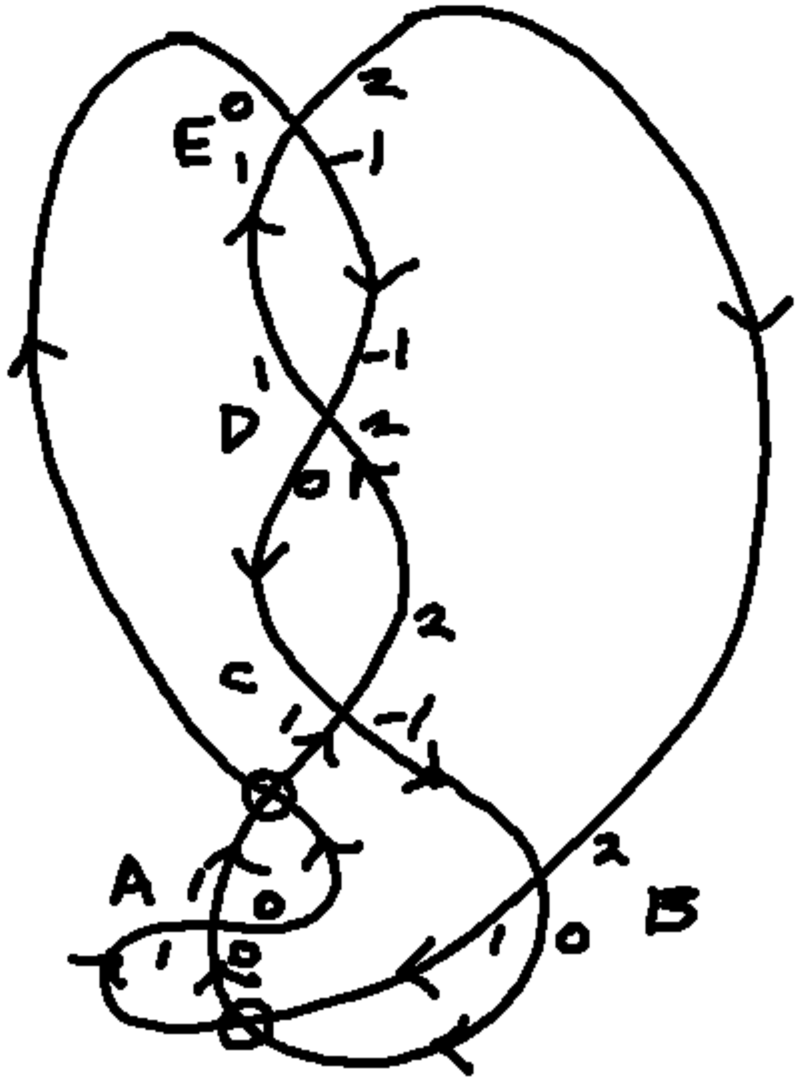
K_0



		w_+	w_-
+	A	0	0
+	B	2	-2
+	C	2	-2
+	D	-2	2
-	E	-2	2

$J_K = 1$

$$P_K = 2t^2 + 2t^{-2} - 4$$

Next example: K with $P_K = 0$ but K nontrivial etc...

How do you compute $\langle K \rangle$?

$$\langle \times \rangle = A \langle \asymp \rangle + A^{-1} \langle)(\rangle$$

$$\langle O K \rangle = \underbrace{(-A^{2} - A^{-2})}_{\delta} \langle K \rangle$$

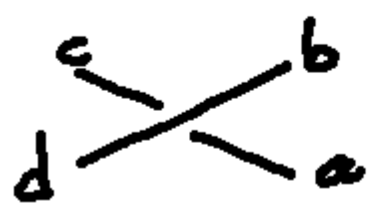
$$\langle O \rangle = 1$$

$\longleftrightarrow$

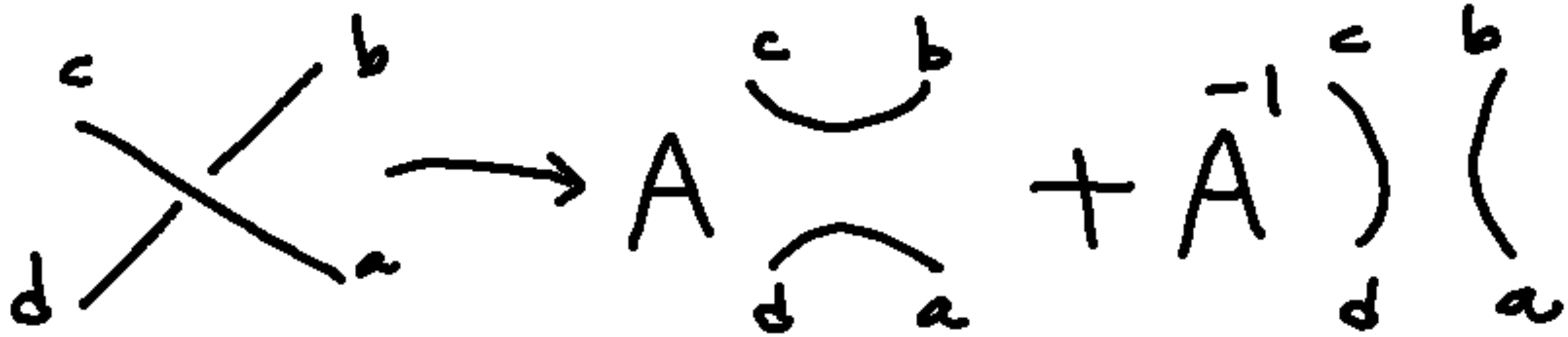
$X[c,d,a,b]$

$\xrightarrow[\text{code}]{} X[a,b,c,d]$

$=$



$\xrightarrow[\text{code}]{} X[b,c,d,a]$



$$X[a,b,c,d] \longrightarrow A\,\delta[b\,c]\,\delta[a\,d] + A^{-1}\delta[c\,d]\,\delta[a\,b]$$

$$\delta(x\,y)\,\delta(y,z) \equiv \delta(x\,z)$$

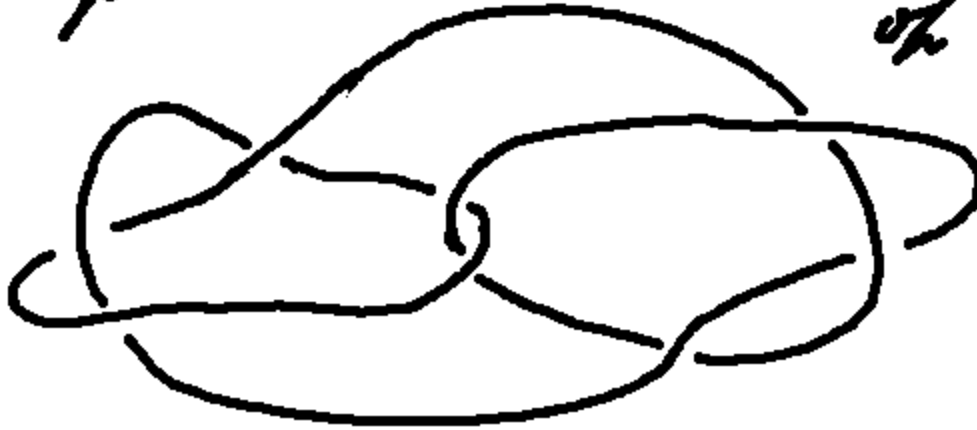
$$\delta(x\,x) \equiv \delta$$

$$\equiv \delta = -A^2 - A^{-2}$$

$X[a, d, f, c]$
$X[e, b, d, a]$
$X[c, f, b, e]$

Mathematica: $X[a, b, f, c] \ldots X[c, f, b, e]$

An example



K has a lot of cancellation in $\langle K \rangle$

$$= +A^{-12} + A^{-4} - A^{8}$$

How does cancellation occur? Can it happen that $J_K = 1$ when K knotted?

Yes. K virtual

Not known K classical

Rewrite: $\langle \times \rangle = \overset{A}{\langle \asymp \rangle} - q\,\overset{B}{\langle)\,(\rangle}$

$$\langle O \rangle = q + q^{-1}$$

(don't normalize $\langle O \rangle = 1$)

$$\langle O\, K \rangle = (q + q^{-1})\langle K \rangle$$

$$\langle K \rangle = \sum_{s \in \text{Enhanced States}} (-1)^{i(s)} q^{j(s)}$$

$i(s) = \#$ of B smoothings

$j(s) = i(s) + \lambda(s)$

$\lambda(s) = \sum \pm$ on enhancement.

e.g.  

$\longrightarrow (-1)^0 q^{0+2} = q^2$

$$\langle K \rangle = \sum_s (-1)^{i(s)} q^{j(s)},$$

$d^{ij} = \dim C^{ij}$

C^{ij} = module gen ~~by~~

$$= \sum_j q^j \sum_i (-1)^i \underbrace{\#\left\{ s \,\middle|\, \begin{matrix} i(s)=i \\ j(s)=j \end{matrix} \right\}}_{d^{ij}}$$

$$= \sum_j q^j \underbrace{\left(\sum_i (-1)^i d^{ij} \right)} = \sum_j q^j \underbrace{\left(\sum_i (-1)^i \dim C^{ij} \right)}_{\chi(C^{\bullet j})}$$

Can we use this to investigate when $d^{ij} \neq 0$.

If $\exists$ chain cx with $\partial : C^{\bullet j} \to C^{\bullet+1\, j}$

$$\partial \circ \partial = \emptyset$$

$$C^{ij} \xrightarrow{\partial} C^{i+1\,j}$$

$$\partial[XYZWA\cdot] \qquad A \xrightarrow{(-1)^{\#A \text{ before}}} B$$

$$\boxed{(-1)^{\#\text{preceding } A\text{'s}}}$$

$$s \rightsquigarrow s'$$

s

$[AAA]$

If s is a new smoothing s', then

$$\partial s = \sum \pm \text{Something related to smoothing each indiv } A.$$

$$\partial[AAA] = [BAA] - [ABA] + [AAB]$$

$$\partial[BAA] = [BBA] - [BAB]$$

$$-\partial[ABA] = -[BBA] + [ABB]$$

$$\partial[AAB] = [BAB] - [ABB]$$

$1\cdot 1 = 1$

$s \qquad\qquad s'$

$$\boxed{C^{i,j} \xrightarrow{\partial} C^{i+1,j}}$$

$$\boxed{\begin{matrix} + \leftrightarrow 1 \\ - \leftrightarrow x \end{matrix}}$$

$i(s) = 0$

$j(s) = i(s) + \lambda(s)$

$$\boxed{\lambda(s) = 2}$$

$i(s') = i(s) + 1$

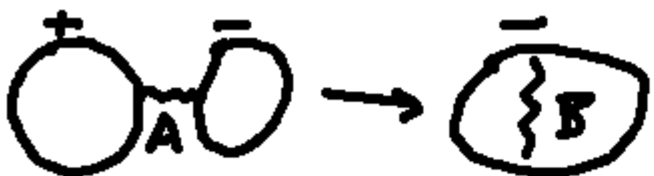
$j(s) \underset{\text{wanted}}{=} j(s') = i(s') + \lambda(s')$

$= i(s) + 1$

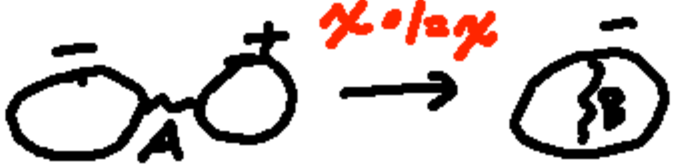
$i(s) + \lambda(s) = i(s) + 1 + \lambda(s')$

$$\boxed{\lambda(s') = \lambda(s) - 1}$$

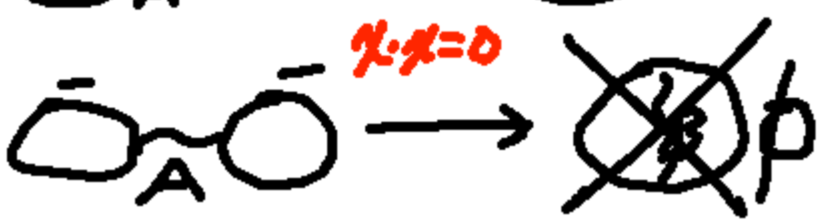
$1 \cdot x = x$



$x \cdot 1 = x$

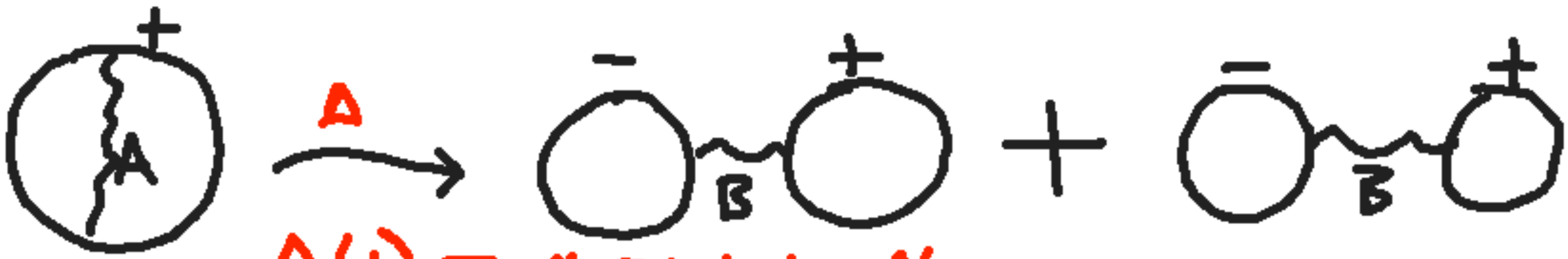


$x \cdot x = 0$



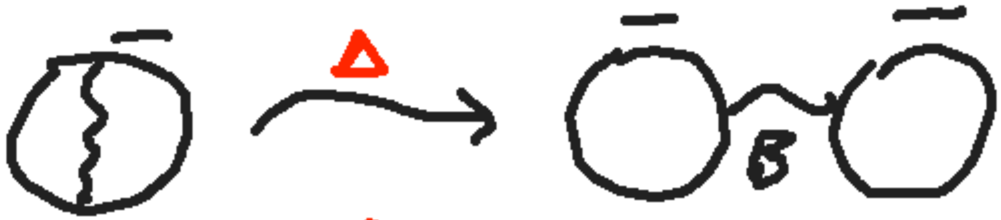
all described by $\mathbb{Z}[x]/(x^2)$

$\| \{n + mx \mid x^2 = 0\}$



$\Delta(1) = x \otimes 1 + 1 \otimes x$

$M(0) = \mathbb{Z}[x]/(x^2)$



$\Delta(x) = x \otimes x$

$m : \text{Module}(0) \otimes \text{Module}(0) \longrightarrow M(0)$

$\Delta : M(0) \longrightarrow M(0) \otimes M(0)$

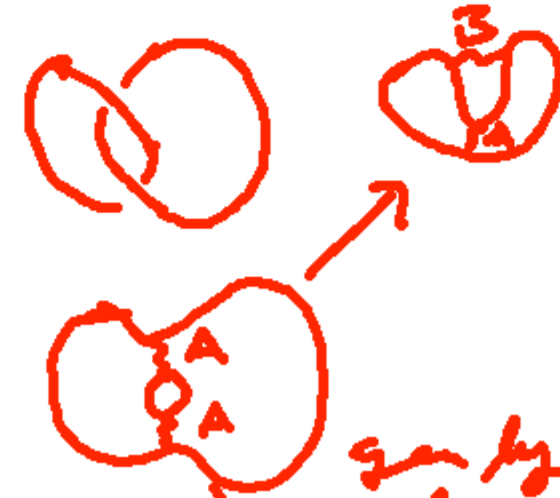
---

underlying $\partial : C^{ij} \longrightarrow C^{i+1\, j}$

Need to see that $\partial \circ \partial \bmod 2 = 0$

If we do not use the signs then . . .

$$
\begin{tikzcd}
& {[BA]} \arrow[dr, "+"] & \\
{[AA]} \arrow[ur, "m \text{ or } \Delta", "+"'] \arrow[dr, "-"'] \arrow[rr, bend left=15, "Top"] \arrow[rr, bend right=15, "Bot"'] & & {[BB]} \\
& {[AB]} \arrow[ur, "+"'] &
\end{tikzcd}
$$



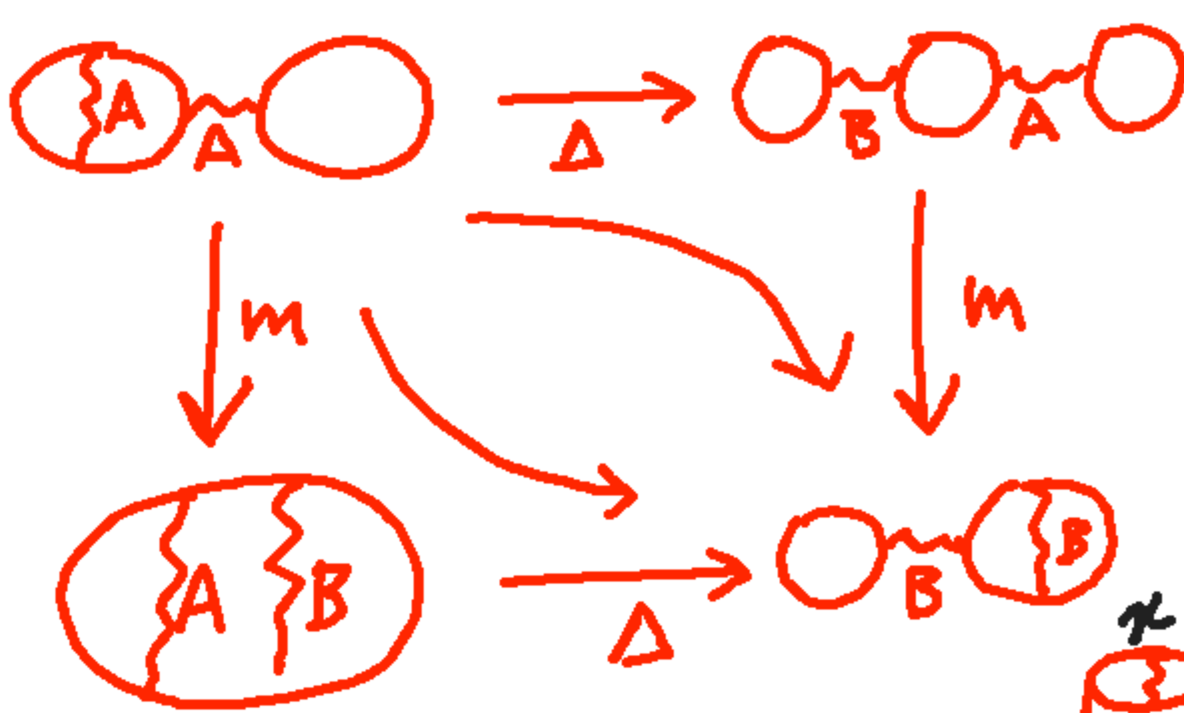
$$Top = -Bot$$

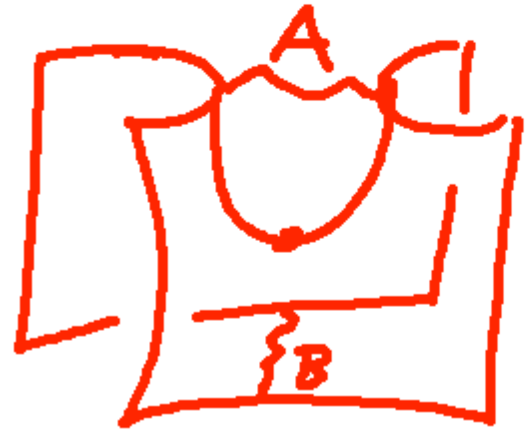
$\Longleftrightarrow$

$$
\begin{tikzcd}
& {[BA]} \arrow[dr] & \\
{[AA]} \arrow[ur] \arrow[dr] \arrow[rr, bend left=15] \arrow[rr, bend right=15] & & {[BB]} \\
& {[AB]} \arrow[ur] &
\end{tikzcd}
$$

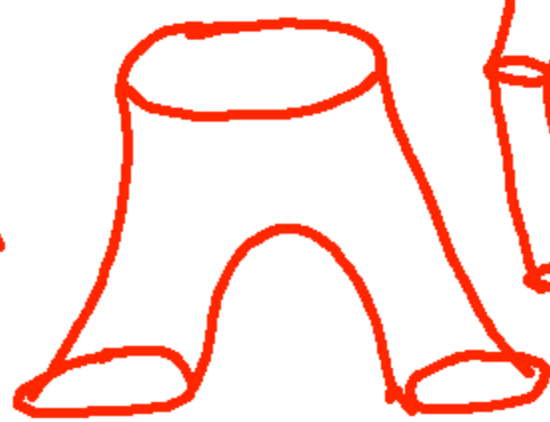
$M\Big($  $\Big)$ = gen by all enhanced states for this

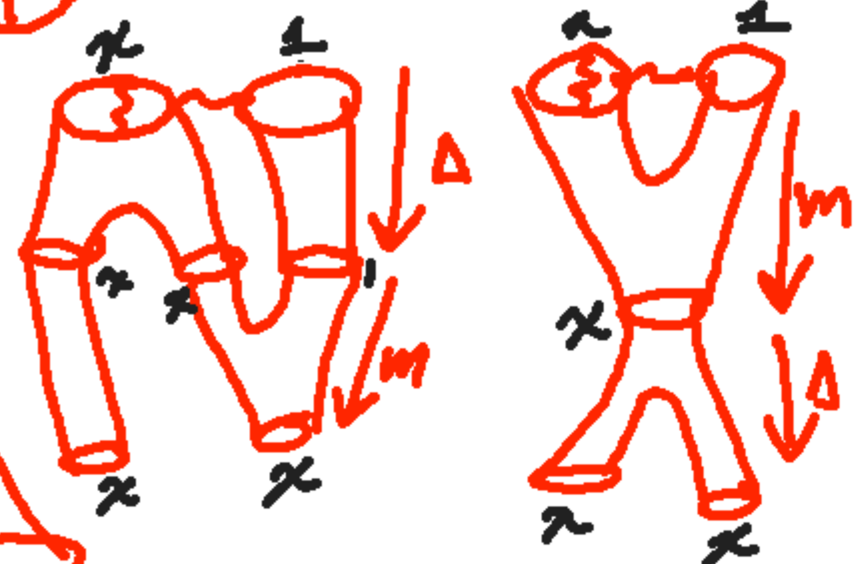
commutes at module level.

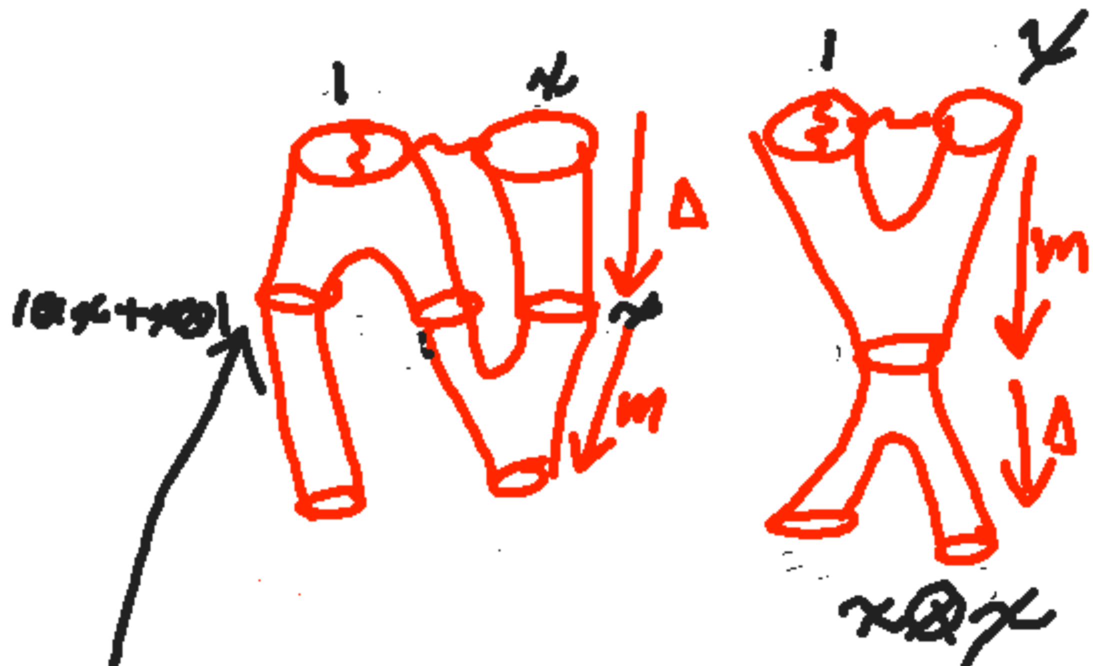

A
A
Δ
B
A
m
m
A
B
Δ
B
B


A
B


m
Δ




x
1
Δ
x
x
m
x
x
1
m
x
Δ

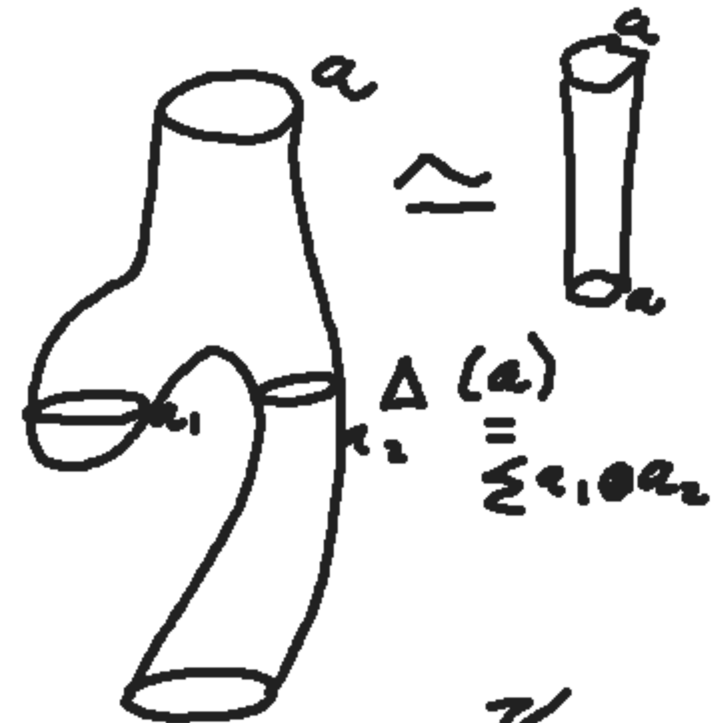


$1\otimes \underbrace{x\otimes x} + x\otimes 1 \otimes \underbrace{x}$

$\downarrow$

$0 + x\otimes x$

x

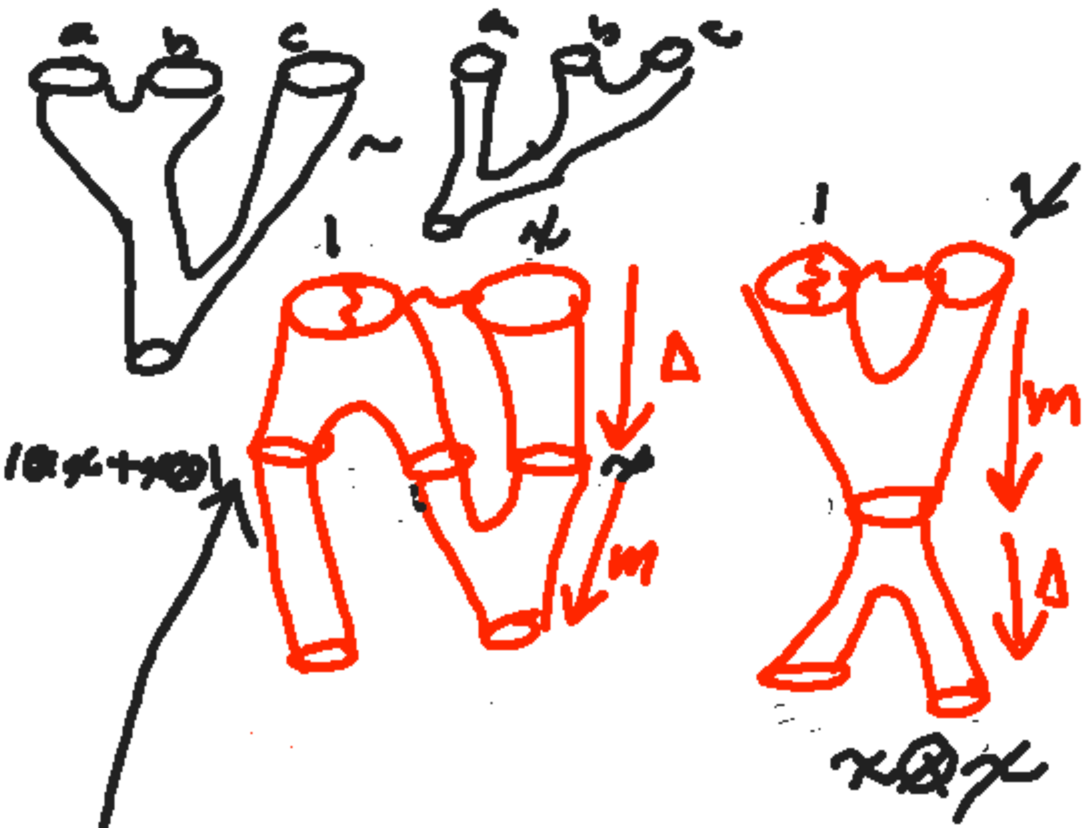


$\simeq$

$\Delta(a) = \sum a_1\otimes a_2$

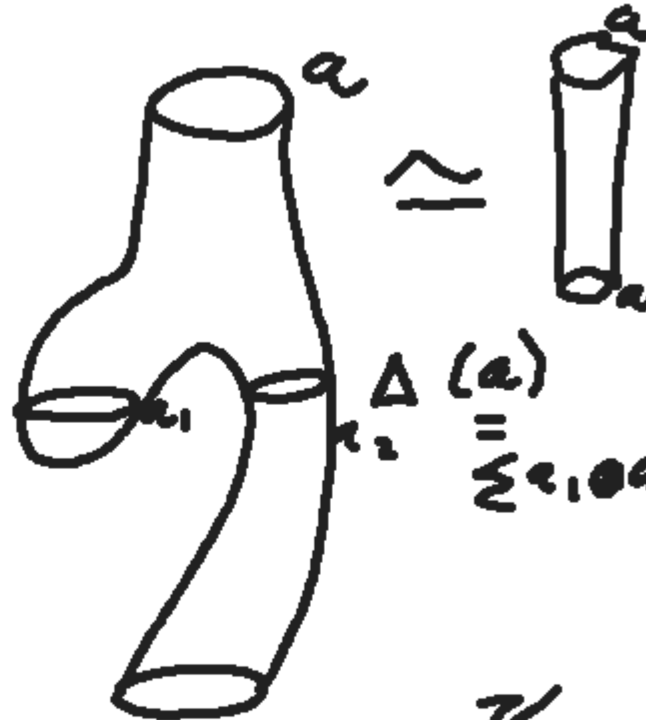
$\varepsilon(a)\in\mathbb{Z}$ $\quad \eta(1) = 1_M$

$\varepsilon(x) = ?,\ \varepsilon(1) = ?$



$$1\otimes x\otimes x + x\otimes 1\otimes x \rightarrow \phi + x\otimes x$$

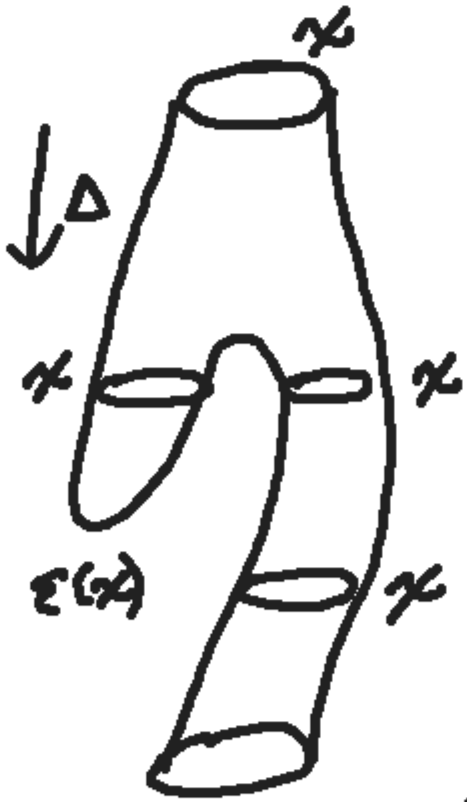
x



$$\Delta(a) = \sum a_1 \otimes a_2$$

$\varepsilon(a) \in \mathbb{Z}$ $\quad$ $\eta(1) = 1_M$

$\varepsilon(x) = ?,\ \varepsilon(1) = ?$



want

$$\varepsilon(x)x = x$$

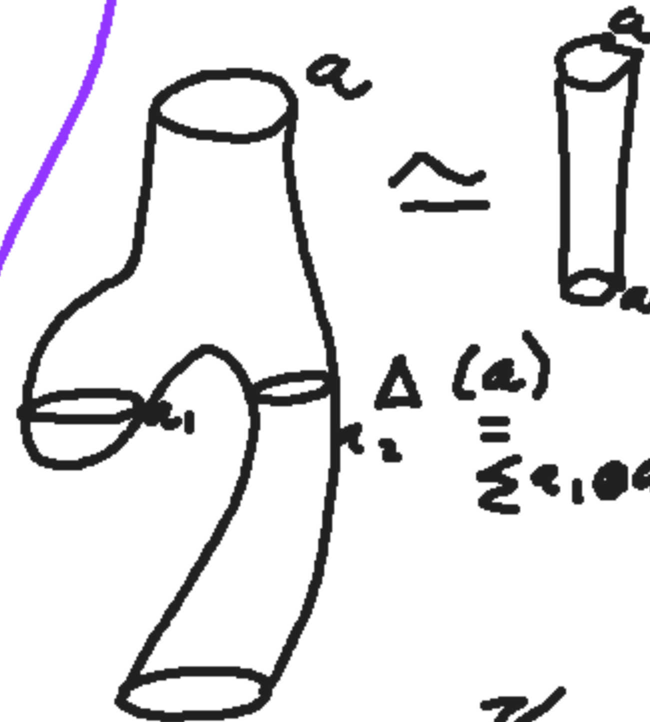
so

$$\boxed{\varepsilon(x) = 1}$$

$x \otimes 1 +$
$1 \otimes x$

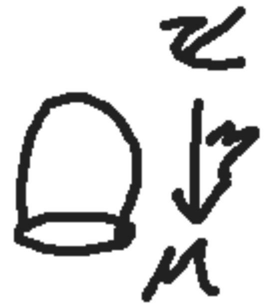
$\varepsilon(x)1$
$+ \varepsilon(1)x$
$\parallel$
$1 + \varepsilon(1)x$

$$\boxed{\varepsilon(1) = 0}$$



$\Delta(a) = \sum a_1 \otimes a_2$

$\varepsilon(a) \in \mathbb{Z}$



$\eta(1) = 1_M$

$\varepsilon(x) = ?,\ \varepsilon(1) = ?$

$\mathcal{A} = \mathbb{Z}[x]/(x^2)$

$\Delta(1) = 1 \otimes x + x \otimes 1$

$\Delta(x) = x \otimes x$

$\eta(1 \in \mathbb{Z}) = 1 \in \mathcal{A}$

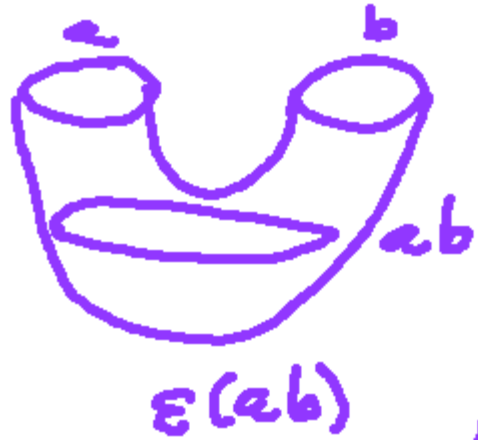
$\varepsilon(x) = 1$

$\varepsilon(1) = 0$

Frobenius algebra

$\mathcal{A} \otimes \mathcal{A} \to \mathbb{Z}$

$\langle \mid \rangle$



$\langle a|b \rangle = \varepsilon(ab)$

$\langle ab|c \rangle = \langle a|bc \rangle$

Joachim Koch

We have at this point defined the chain cx $C^{ij}(K)$

• RM

• Virtuals

$\therefore$: $H^{ij}(K)$ Khovanov Homology.