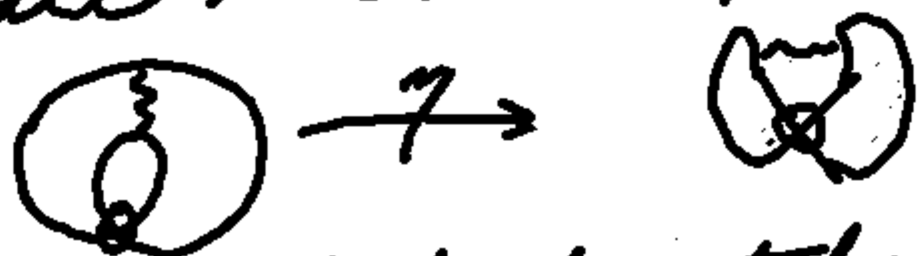
Lecture 12

Rushworth Doubled Kho Homology

Soln to Kho Homology Virtuals / $\mathbb{Z}$.

Recall the basic problem:



We decided to take $\eta = 0$

& adjust accordingly.

Recall what we need to examine.

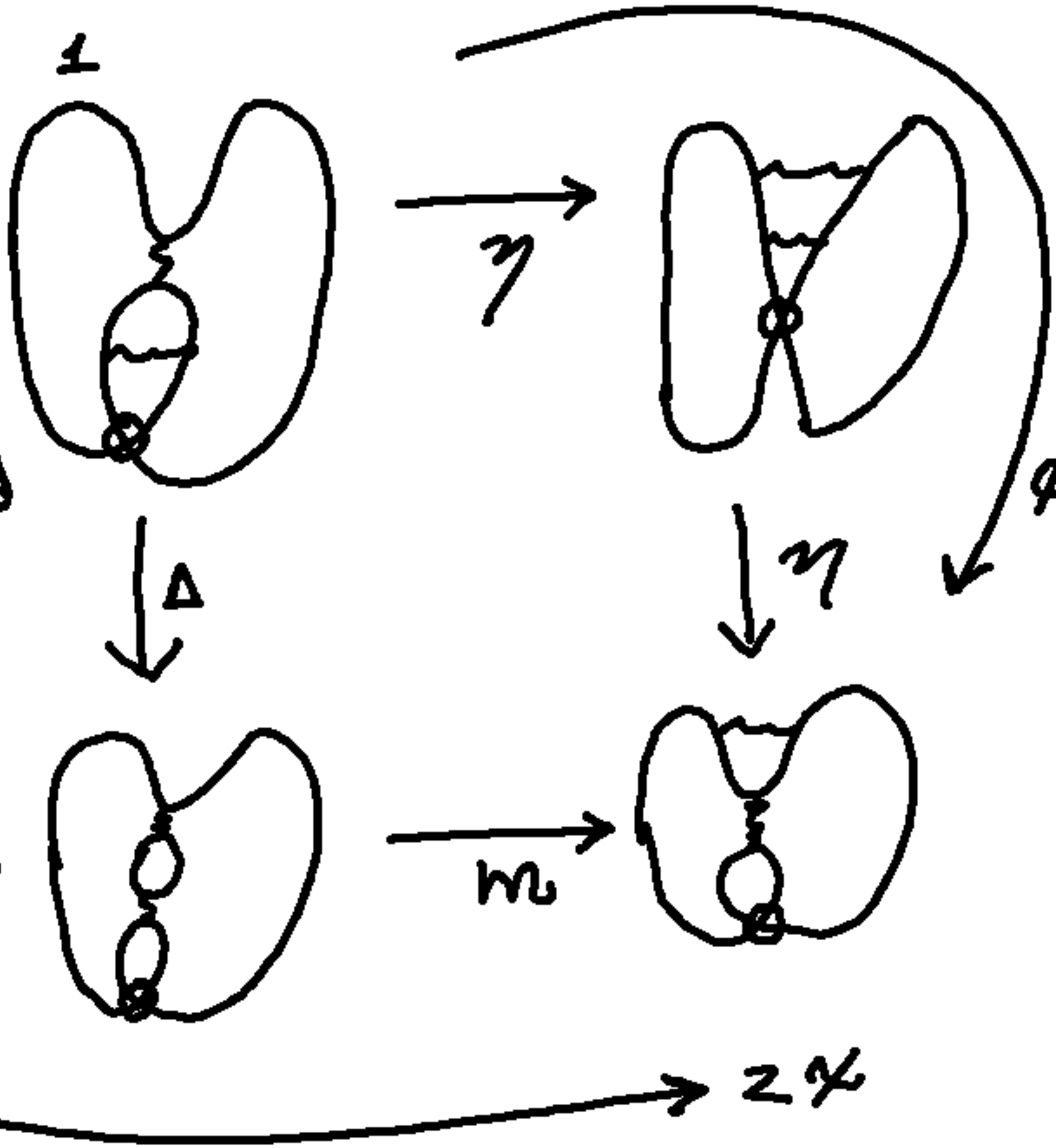
usual Kho alg

$x^2=0$

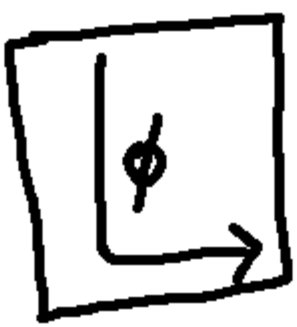
$\Delta(x)=x\otimes x$

$\Delta(1)=1\otimes x+x\otimes 1$

We changed alg rules to local coeffs made ...



$1\otimes x+x\otimes 1$



Rushworth

Double Module V

So we have

$V^u \oplus V^\ell$

two copies.

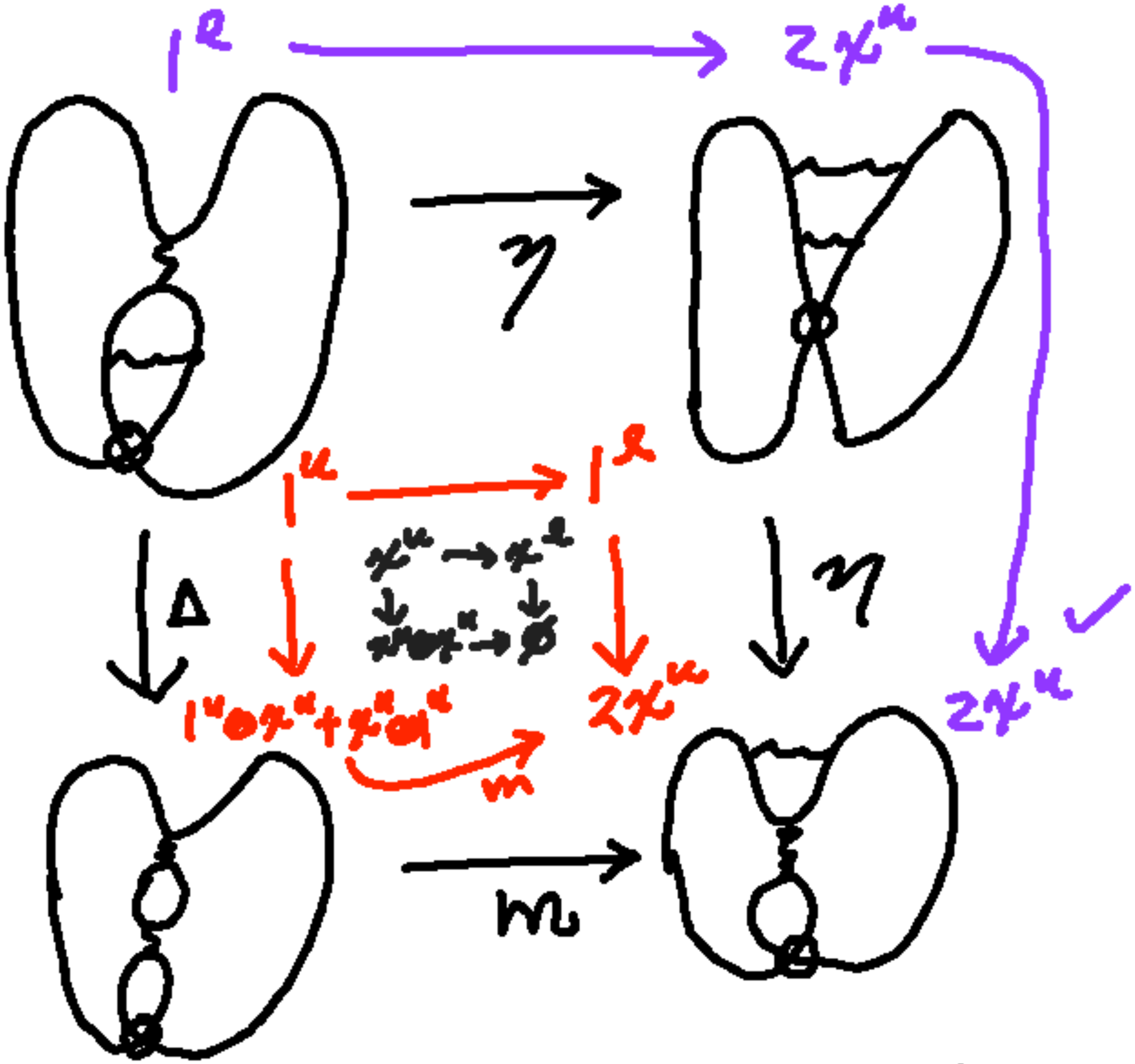
$m, \Delta : V^u \otimes V^u$

$V^\ell \otimes V^\ell$

as usual.

η moves between the modules.

$$\boxed{\begin{aligned} \eta(1^u) &= 1^\ell \\ \eta(x^u) &= x^\ell \\ \eta(1^\ell) &= 2x^u \\ \eta(x^\ell) &= 0 \end{aligned}}$$



Use usual sign conventions for the homology.

Rushworth

Double Module V

So we have

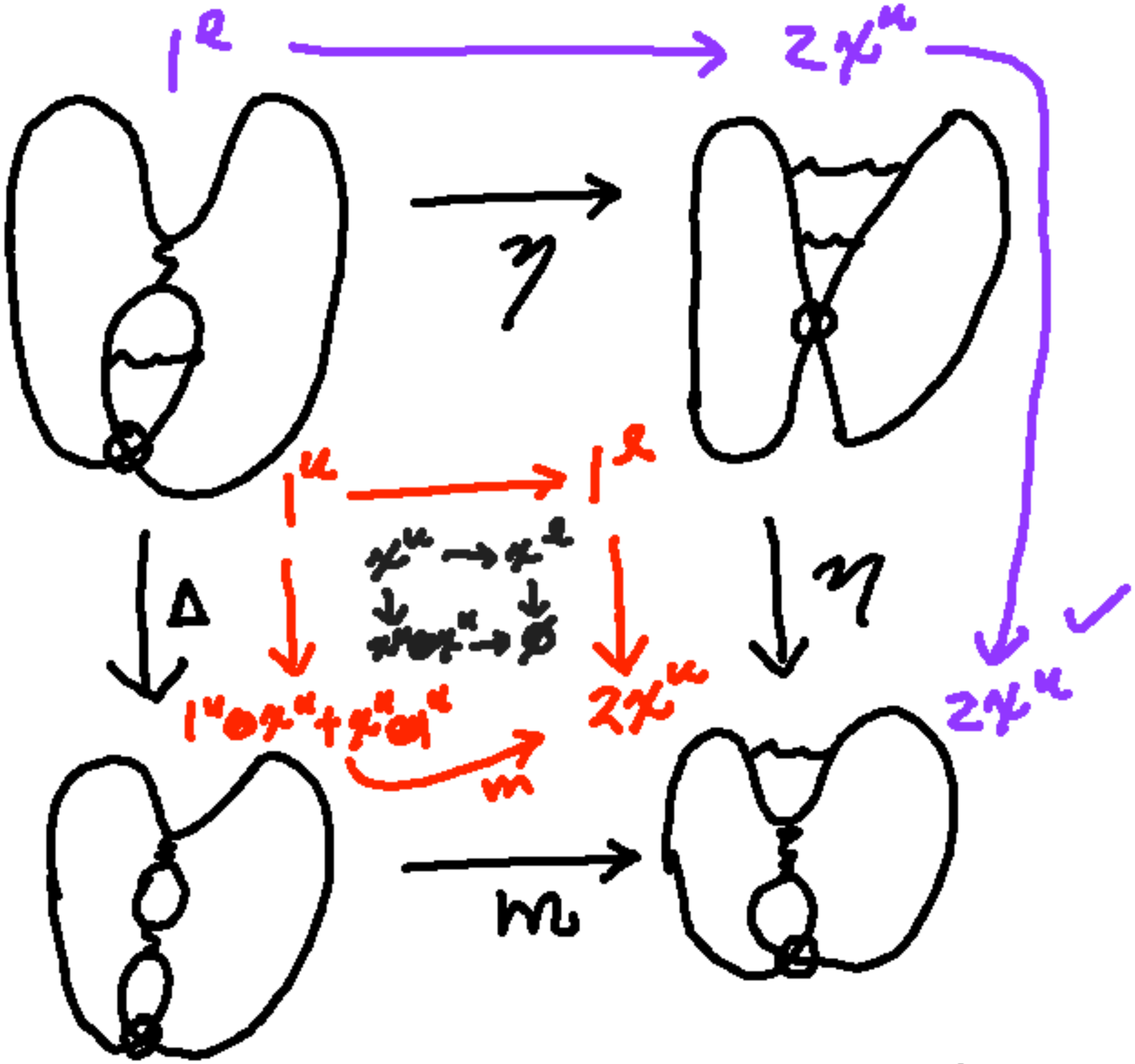
$V^u \oplus V^l$

two copies.

$m, \Delta : V^u \otimes V^u$
$V^l \otimes V^l$
as usual.

η moves between the modules.

$$\boxed{\begin{aligned} \eta(1^u) &= 1^l \\ \eta(x^u) &= x^l \\ \eta(1^l) &= 2x^u \\ \eta(x^l) &= 0 \end{aligned}}$$



Use usual sign conventions for the homology.

Lee Algebra

$$x^2 = 1$$
$$\Delta(1) = 1 \otimes x + x \otimes 1$$
$$\Delta(x) = x \otimes x + 1 \otimes 1$$
$$\varepsilon(x) = 1, \ \varepsilon(1) = 0$$

Rushworth η

$$\eta(1^u) = 1^l$$
$$\eta(x^u) = x^l$$
$$\eta(1^l) = 2x^u$$
$$\eta(x^l) = 2\,1^u$$

$$\begin{tikzcd}
\bullet \arrow[r, "\eta"] \arrow[d, "\Delta"'] & \bullet \arrow[d, "\eta"] \\
\bullet \arrow[r, "m"'] & \bullet
\end{tikzcd}$$

$$\begin{tikzcd}
1^u \arrow[r] \arrow[d] & 1^l \arrow[d] \\
1 \otimes x^u + x^u \otimes 1 \arrow[r] & 2x^u
\end{tikzcd}$$

$$\begin{tikzcd}
x^u \arrow[r] \arrow[d, "\Delta"'] & x^l \arrow[d] \\
x^u \otimes x^u + 1^u \otimes 1^u \arrow[r, "m"] & 2\,1^u
\end{tikzcd}$$

Lee Algebra

$$x^2 = 1$$
$$\Delta(1) = 1 \otimes x + x \otimes 1$$
$$\Delta(x) = x \otimes x + 1 \otimes 1$$
$$\varepsilon(x) = 1, \quad \varepsilon(1) = 0$$

$$r = (1+x)/2, \quad g = (1-x)/2$$

$$g^2 = g, \quad r^2 = r$$
$$rg = gr = 0$$
$$r + g = 1$$
$$\Delta(r) = 2r \otimes r$$
$$\Delta(g) = -2g \otimes g$$

Rushworth η

$$\eta(1^u) = 1^\ell$$
$$\eta(x^u) = x^\ell$$
$$\eta(1^\ell) = 2x^u$$
$$\eta(x^\ell) = 2 1^u$$

$$\Longrightarrow$$

$$\eta(r^u) = r^\ell$$
$$\eta(g^u) = g^\ell$$
$$\eta(r^\ell) = 2r^u$$
$$\eta(g^\ell) = -2g^u$$

$$\checkmark \quad \begin{tikzcd} r^u \arrow[r, "\eta"] \arrow[d, "\Delta"] & r^\ell \arrow[d, "\eta"] \\ 2r^u \otimes r^u \arrow[r, "m"'] & 2r^u \end{tikzcd}$$

$$\eta(r^u) = \eta\left(\frac{1^u + x^u}{2}\right)$$

$$= \frac{1^\ell + x^\ell}{2}$$

$$\underline{\eta(r^u) = r^\ell}$$

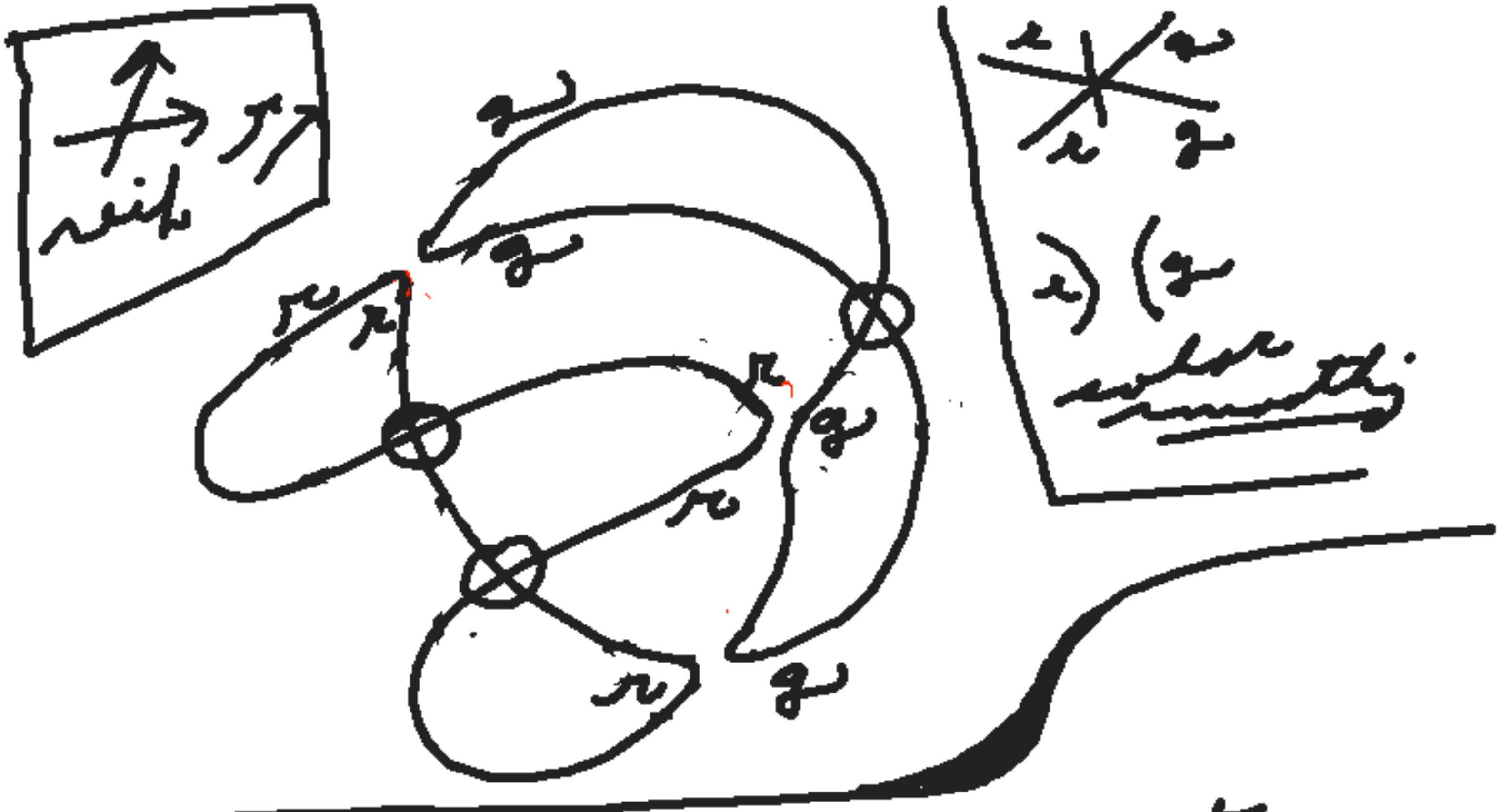
<u>exercise</u>: check <u>the rest</u>

$$\eta(\kappa^u) = \eta\left(\frac{1^u + x^u}{2}\right)$$

$$= \frac{1^\ell + x^\ell}{2}$$

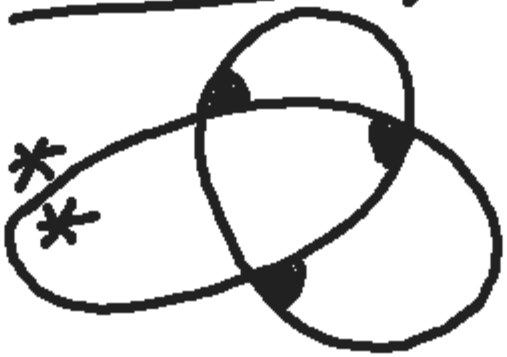
$$\underline{\eta(\kappa^u) = \kappa^\ell}$$

<u>exercise</u>: check <u>the rest</u>



Continued next week.

Marker States



<u>one</u> marker per region

Regions = Nodes + 2

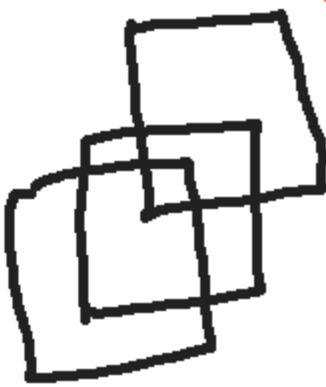
smooth

$\longrightarrow$ smooth

single Jordan curve

1–1 correspondence between

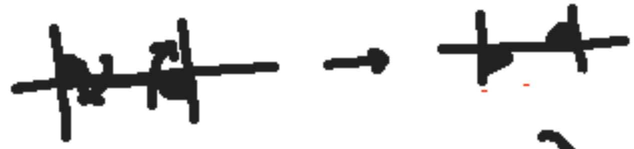
Marker States (choice of *'s) $\rightleftarrows$ Jordan Euler trails on the Diagram.

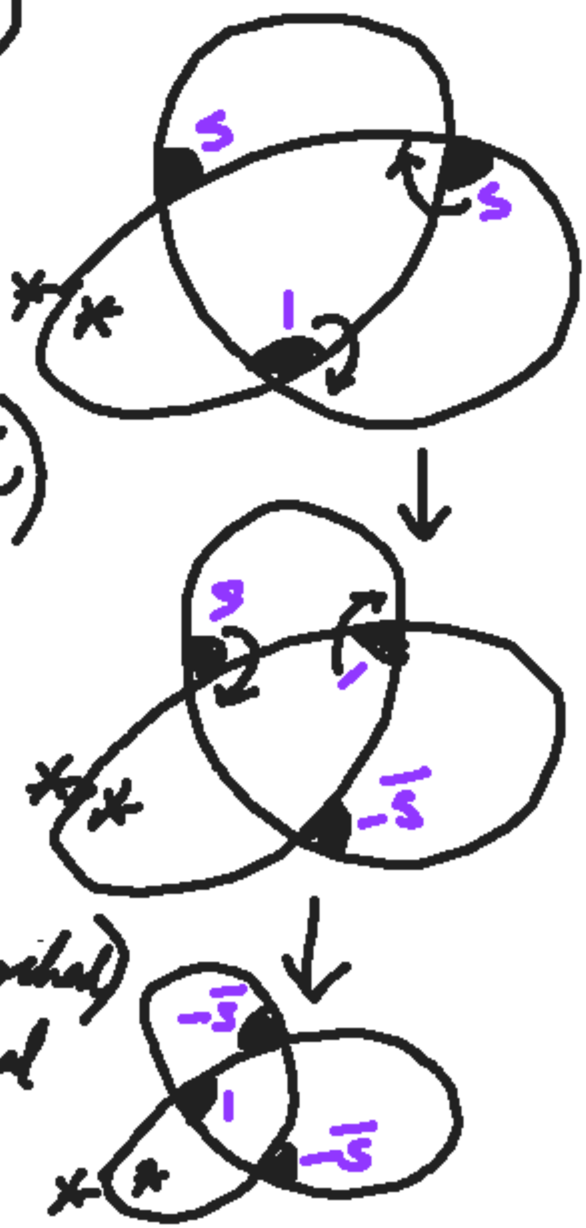


Carroll

clocked state (only clockwise moves)

(counterclocked) anticlocked state





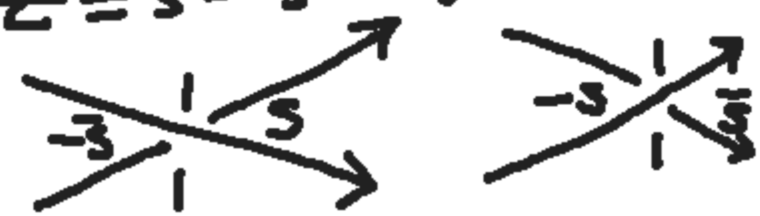
Conway Polyn

$\nabla_{L_+} - \nabla_{L_-} = z\nabla_{L_0}$

$\nabla_{O} = 1$

Invar R1,2,3

$z = s - s^{-1}$, $\bar{s} = s^{-1}$



$\nabla_K = \sum_S \langle K|S \rangle$

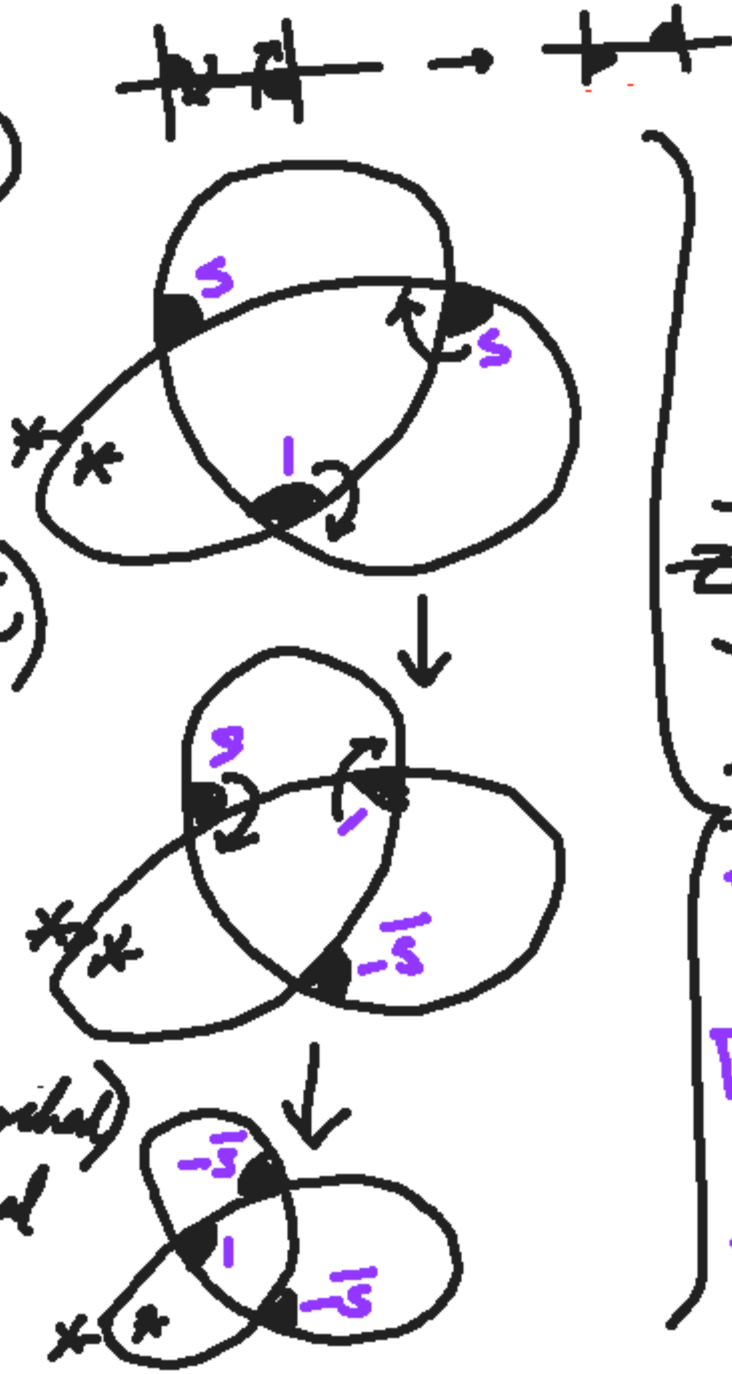
$\nabla_K = s^2 - 1 + s^{-2}$

$= (s - s^{-1})^2 + 1$

$\nabla_K = z^2 + 1$

clocked state (only clockwise moves)

(counterclocked) anticlocked state



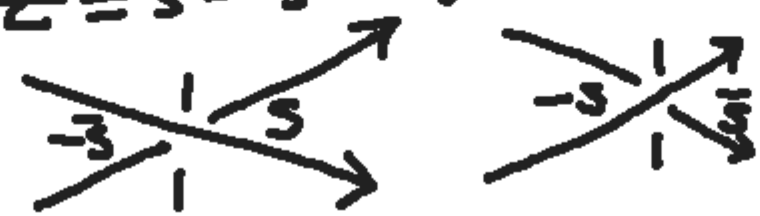
Conway Polyn

$$\nabla_{\nearrow\!\!\!\!\searrow} - \nabla_{\searrow\!\!\!\!\nearrow} = z\,\nabla_{\rightrightarrows}$$

$$\nabla_{O} = 1$$

Invar R1,2,3

$z = s - s^{-1}$, $\bar{s} = s^{-1}$

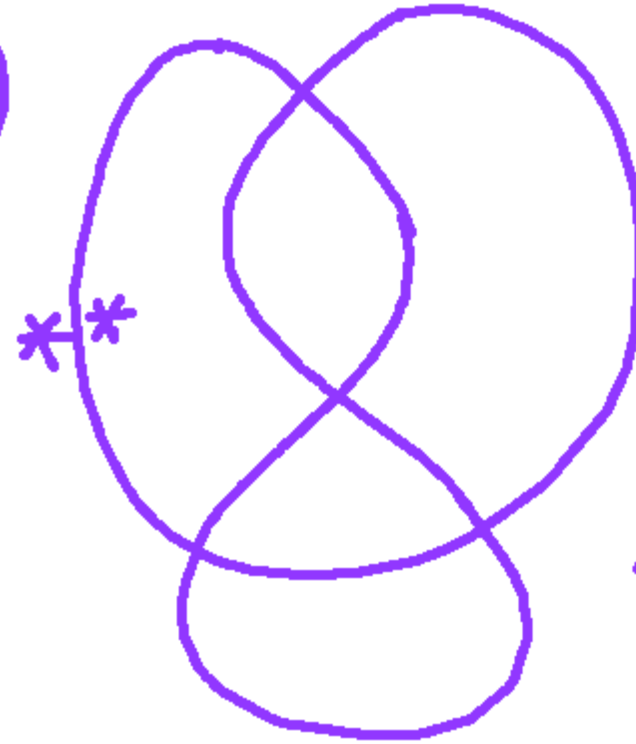


$$\nabla_K = \sum_{S} \langle K|S\rangle$$

$$\nabla_K = s^2 - 1 + s^{-2}$$

$$= (s - s^{-1})^2 + 1$$

$$\nabla_K = z^2 + 1$$



D

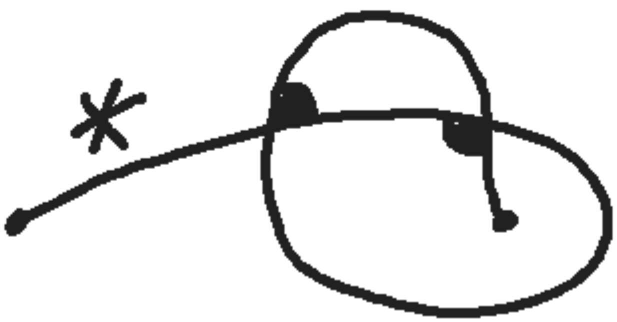
Exercise

Find out everything about unknotting for D

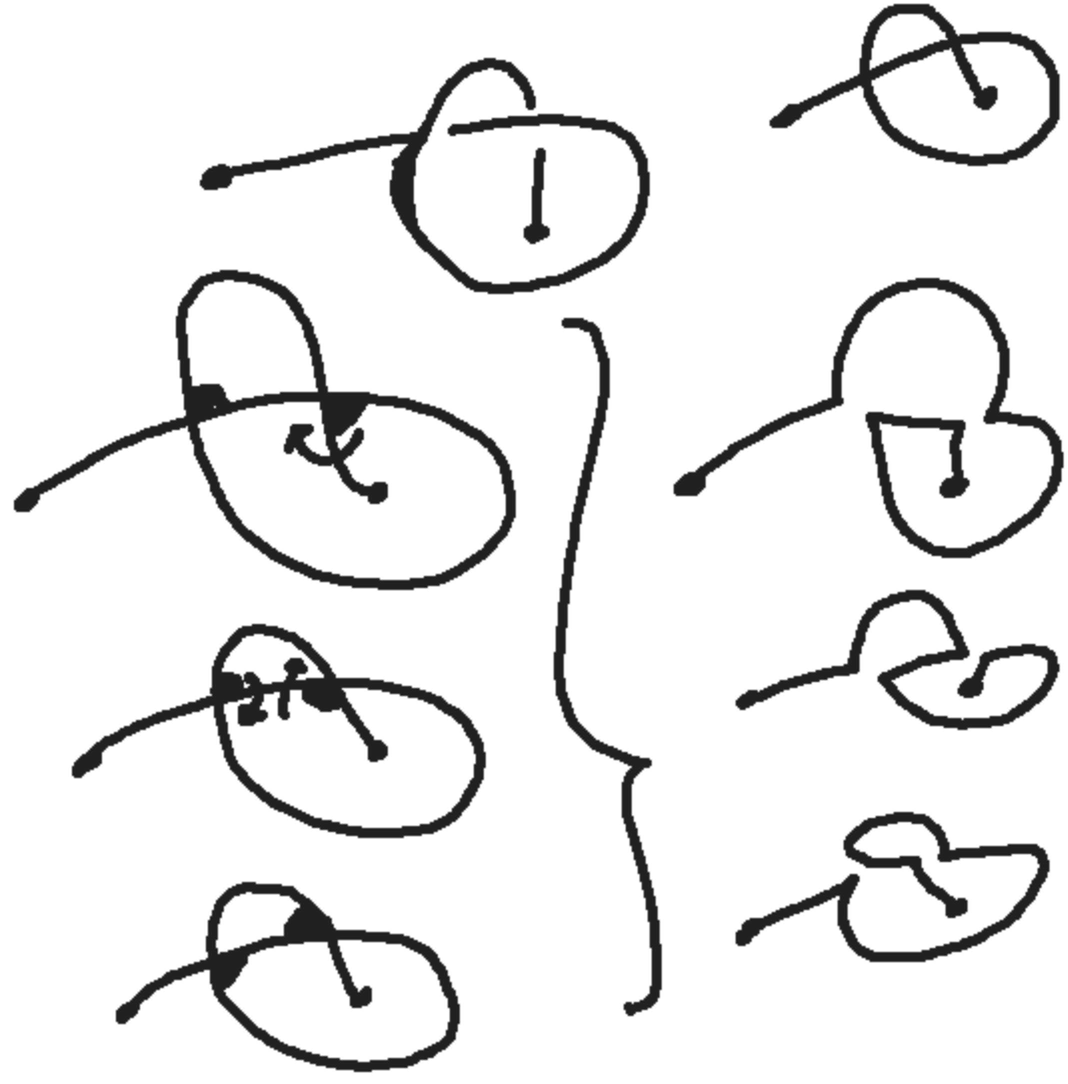
$\forall$ $\nabla_E(s)$ for  E.

Joint work with Neslihan Gügümcü.

Knotoid Marker States

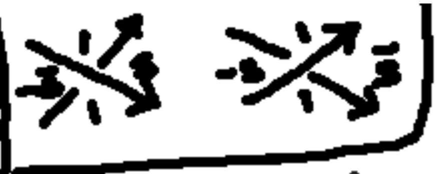


one * region



add

single turns

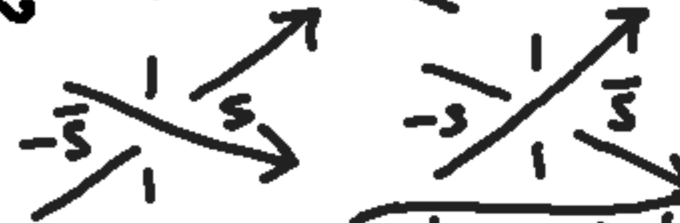


$\Longleftarrow$

$\nabla_{K^*}(s) = \nabla_K(s^{-1})$

$\vec{K} = K.$

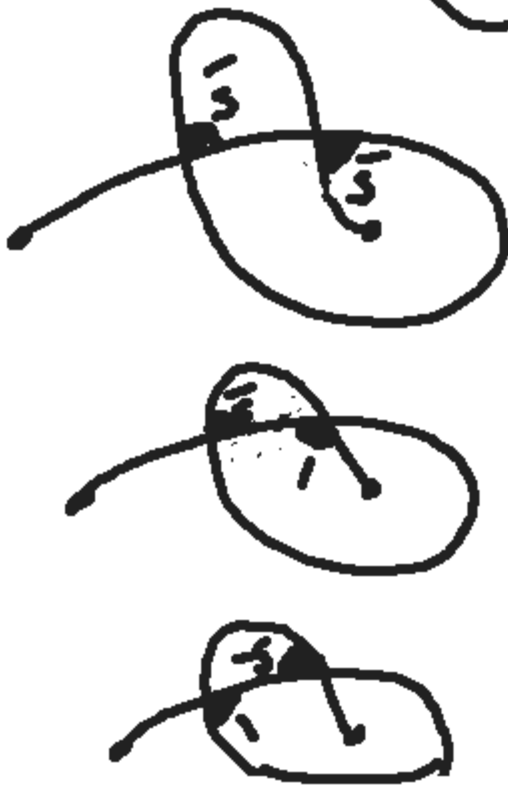
$\nabla_K(s) = \sum \langle K|S \rangle$



$\nabla_{\overleftarrow{K}}(s) = \nabla_{\vec{K}}(-s^{-1})$

$K \longmapsto K^*$

$\nabla_{K^*}(s) = \nabla_K(s^{-1})$



$\nabla_K = s^{-2} - s^{-1} - s$

Thus here $K \neq K^*$ and $\vec{K} \neq \overleftarrow{K}$

Invariant but skein relation $\nabla_{\nearrow\!\!\!\searrow} - \nabla_{\searrow\!\!\!\nearrow} = (s - s^{-1})\nabla_{\rightrightarrows}$ not in general true.

$\overleftarrow{K}$

K^*

$\nabla_{K^*} = s^2 - s - s^{-1}$