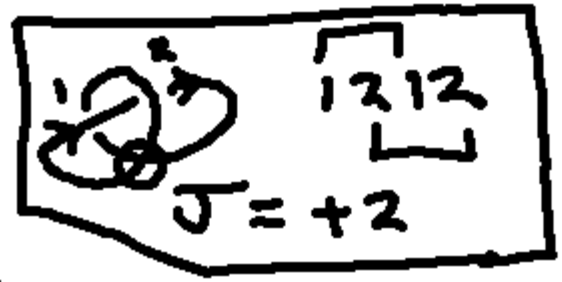
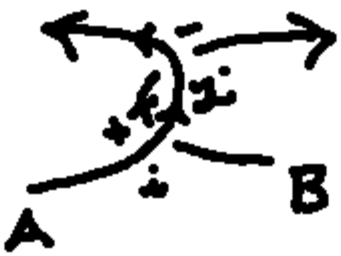
Odd Writhe



$$J_K = \sum_{c \in \text{OddCrossings}(K)} \text{sgn}(c)$$

Show J_K is an invar for VKT.

1.  ← even ✓

2. 

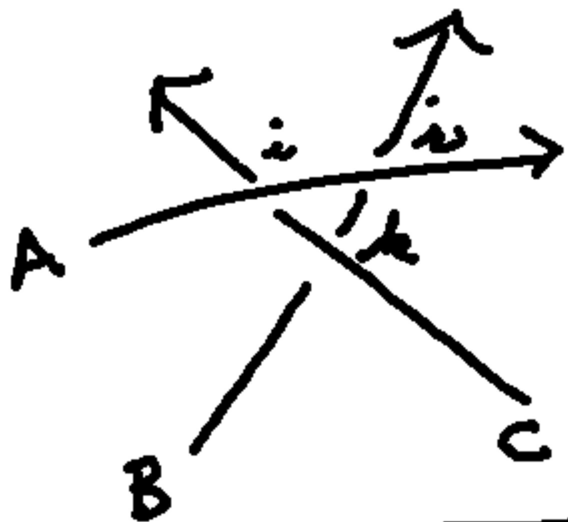
$\alpha\, i\, j\, \beta\, i\, j\, \gamma$

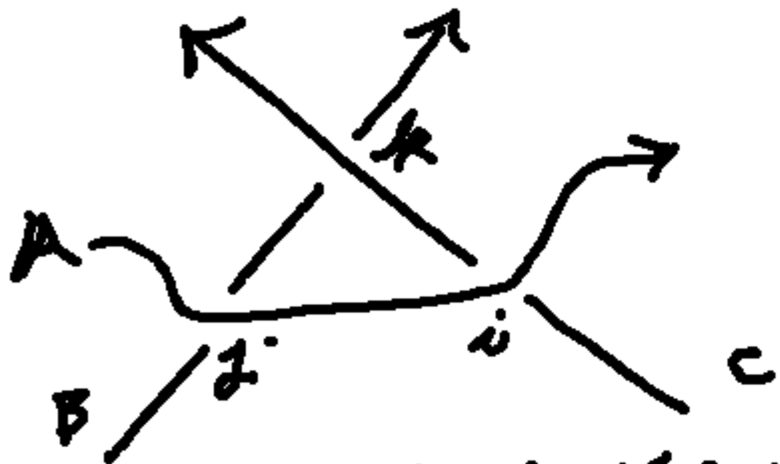
$1 + |\beta|$ $\quad 1 + |\beta|$

both odd or both even
contrib $+1 - 1 = 0$ ✓

$\Downarrow$

J_K invar under R2.





$\alpha j i \beta j k \gamma i k \delta$

$\alpha i j \beta k j \gamma k i \delta$

i: $|\beta| + |\gamma|$
j: $1 + |\beta|$
k: $1 + |\gamma|$

} 1 even, 2 odd or all even } ⇒ Invariance for R3.

---

detour does not affect JK.

∴ JK invar of VK.

Manturov Parity Bracket

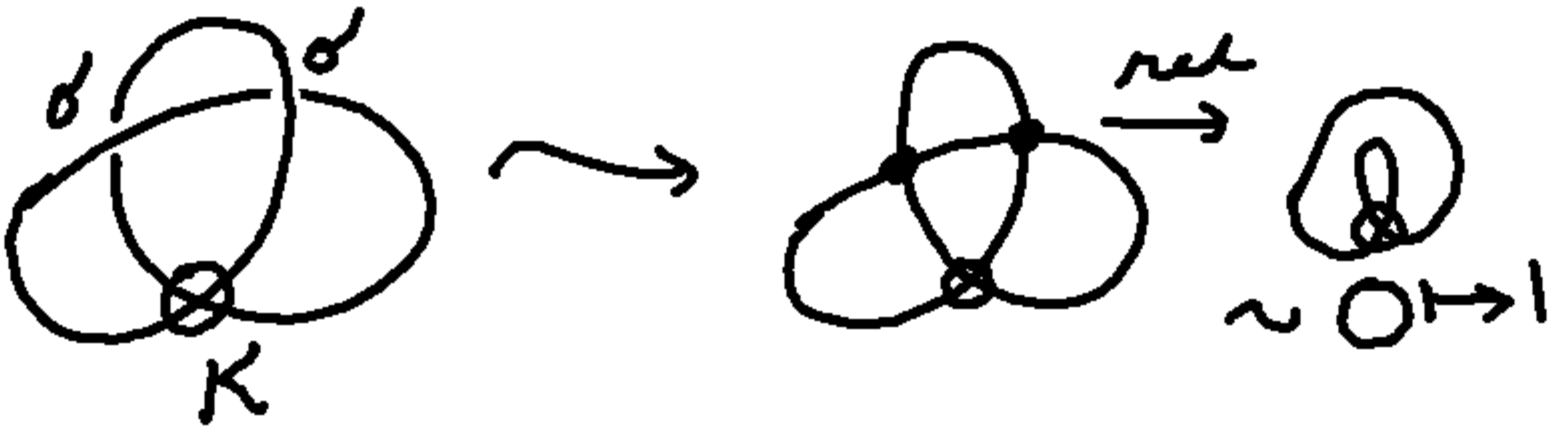
1) Nodify all odd crossings.

2) Expand using  on remaining crossings.

You get a sum of coeffs × graphical loops.

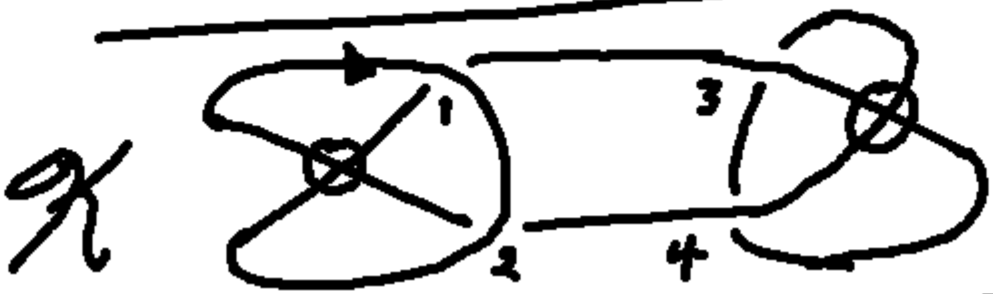
- Evaluate ○ (no nodes) $\longrightarrow \delta = -A^{2} - A^{-2}$.
- Reduce Graphs 

e.g. 



no odd crossings

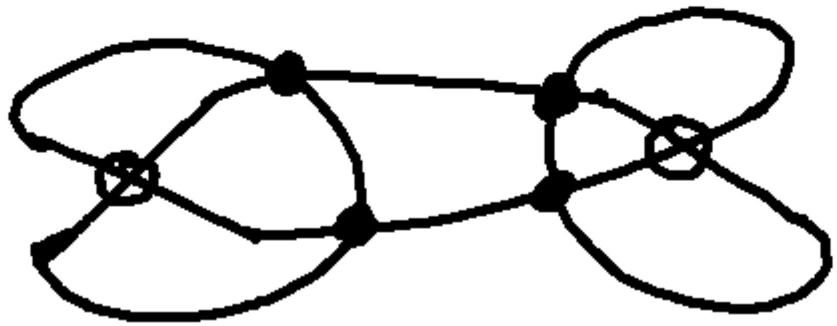
$\not\Rightarrow$ $[k]_p = \langle k \rangle$



1 2 1 3 4 3 4 2 all odd!

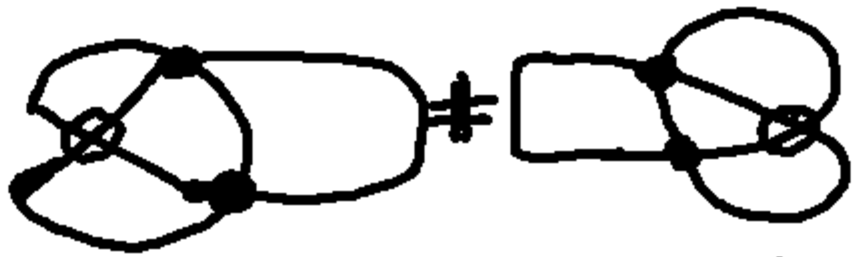
Kishino Diagram < trivial parallel / trivial groups / trivial <>

$f_K = 1.$



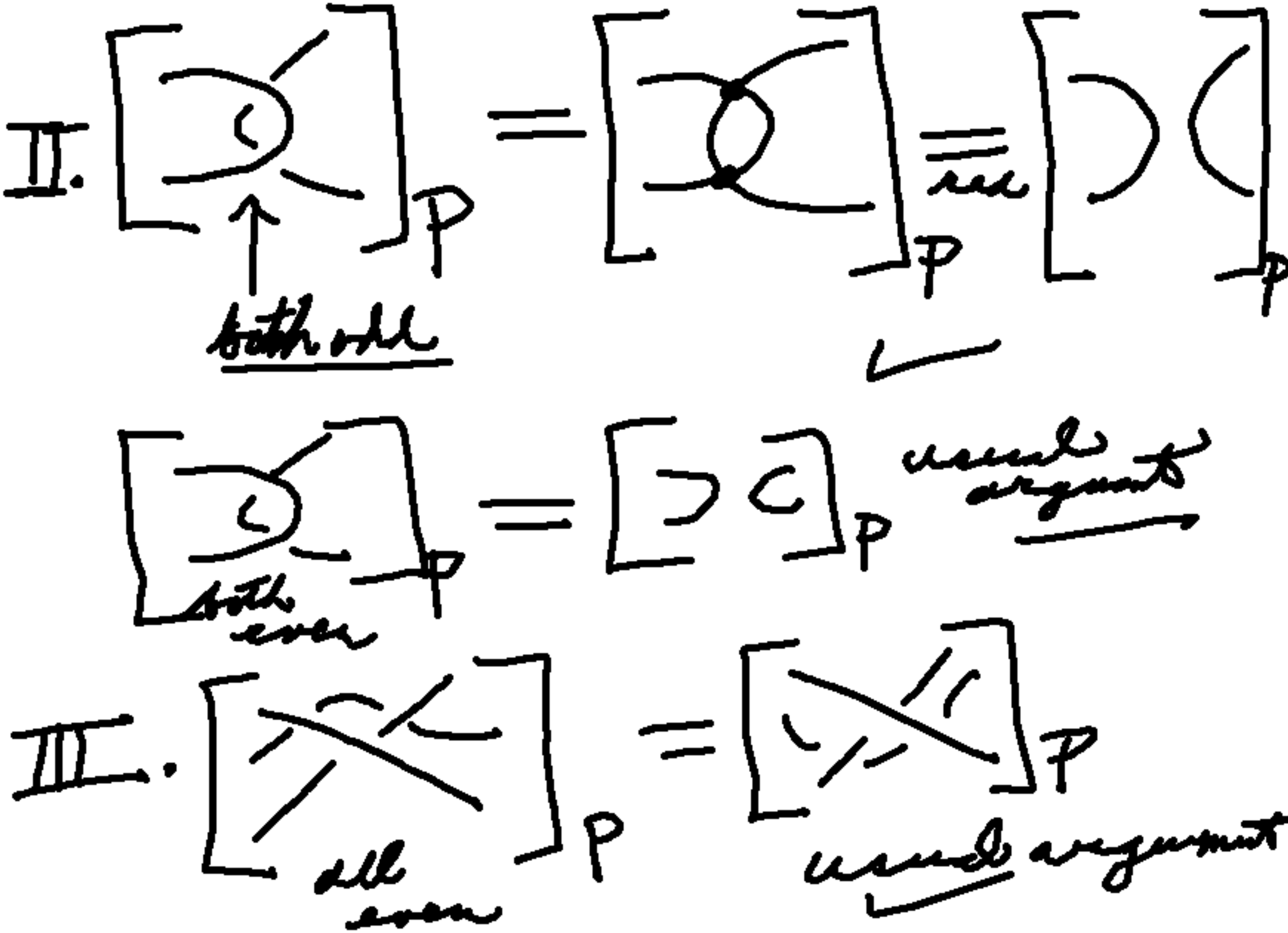
modified
K is shown.

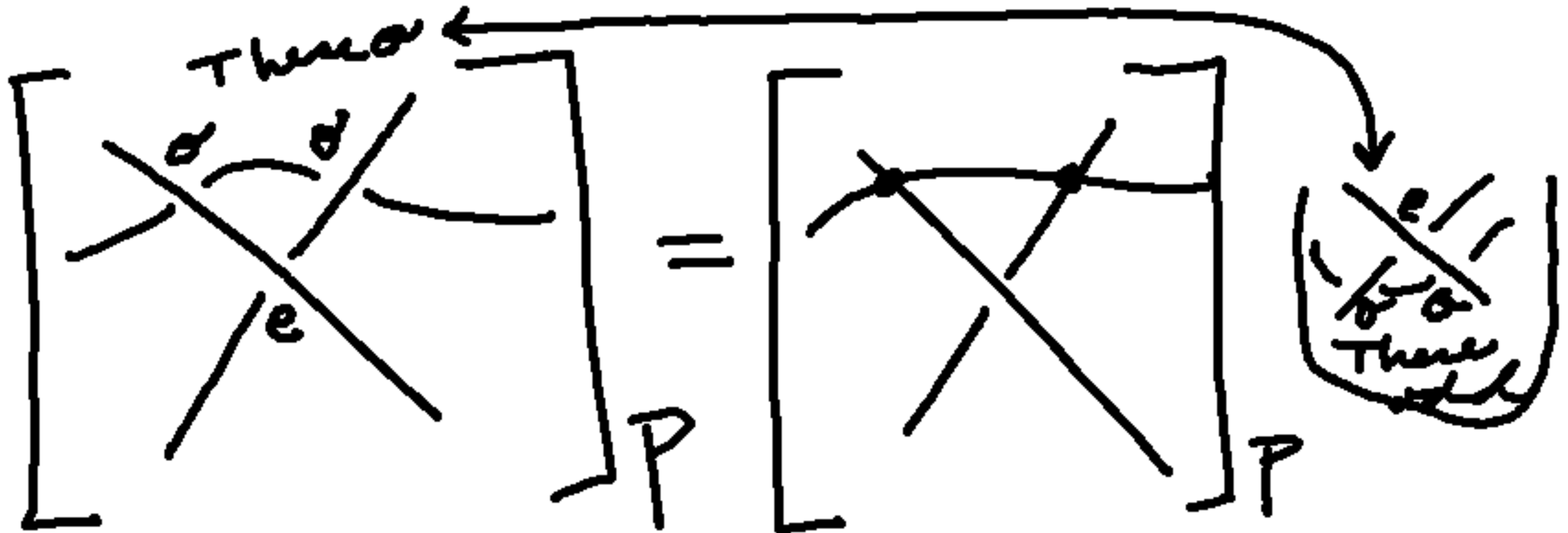
no reductions are available.



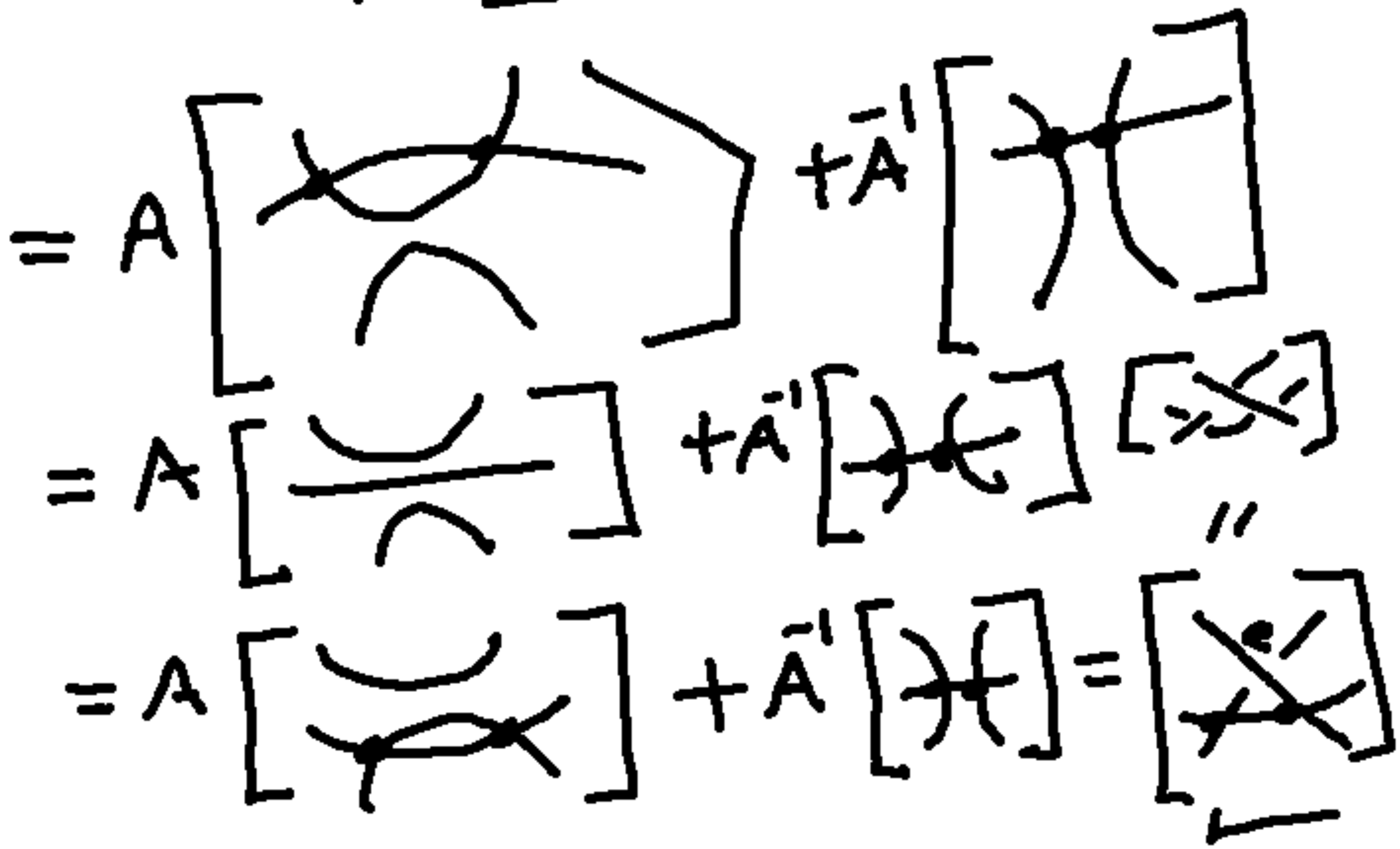
$[K]_p$ = Modified(K) is not trivial
∴ K is a non-trivial
virtual knot.

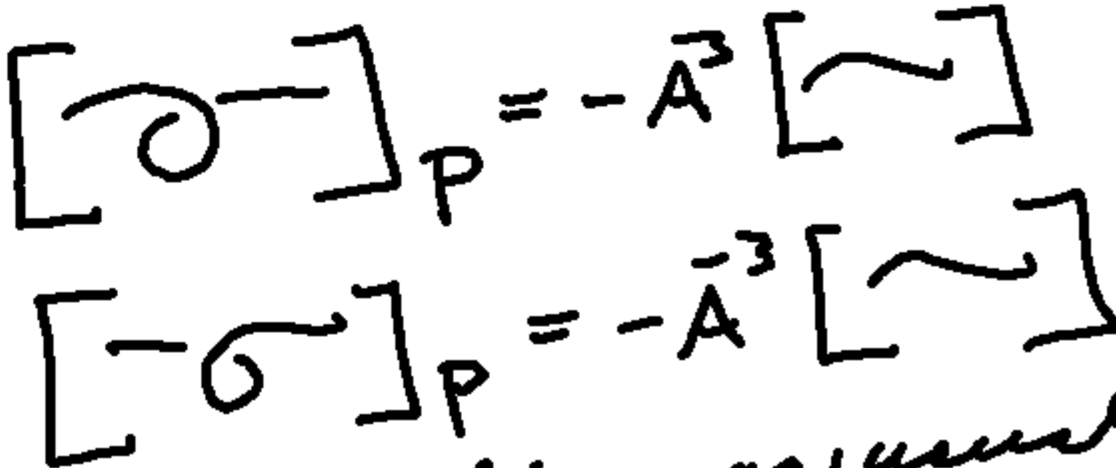
<u>Exercise</u>. Try out the $[\;]_p$ on
other examples.

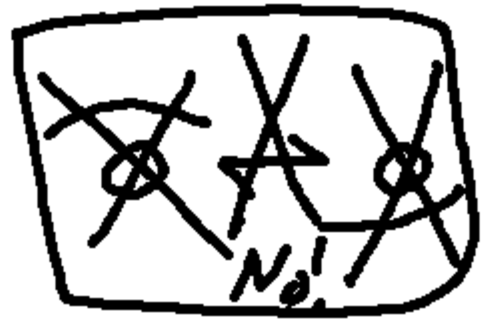

II.
both odd
usual argument
both even
III.
all even
usual argument



<u>w.l.o.g.</u>







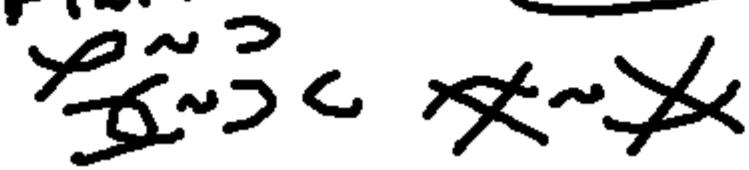
normalize as usual.

Idea: Search for invariants so that an appropriate "picture" of the knot or link is the invariant.

Parity Bracket often applies to flat virtuals. e.g.

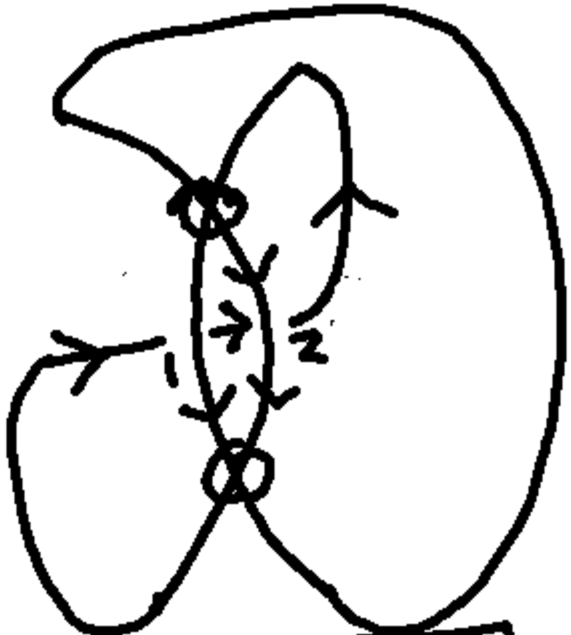
(let $A = A^{-1} = 1$, $\delta = -2$)

1. Flat RM



2.  detour over. But not vice versa.

K

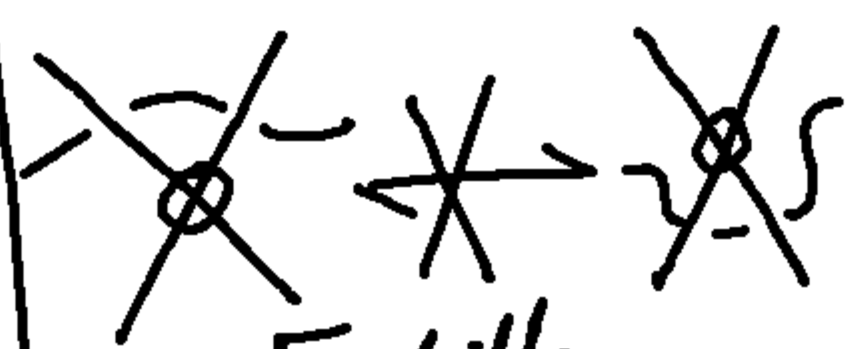


1212

$J_K = ?$

$J_K = 2$

K non-trivial virtual



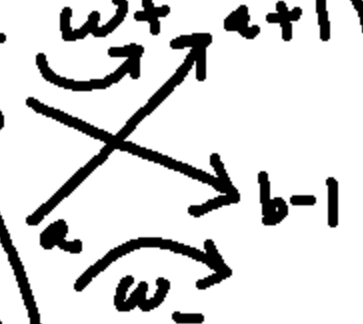
Forbidden

$K \sim O$ via using Forbidden moves.

$\Longrightarrow$ Forbidden Moves is not a consequence of our given VKT moves.

ex: K above $\widetilde{\text{Virtual moves}}$ 

Affine Index Polynomial



$w_+ = b-a-1$

$w_- = a-b+1$

$w_- = -w_+$

	w_+	w_-
A	-2	2
B	2	-2
C	0	0

$$P_K = \sum_{c \in \text{crossings}} \text{sgn}(c)\, t^{W(c)} - \text{writhe}(K)$$

$$= \sum_{c \in Cr(K)} \text{sgn}(c)\left(t^{W(c)} - 1\right)$$

$W(c) = W_{\text{sgn}(c)}$

$$P_K = t^{-2} + t^{2} - t^{0} - (1)$$

$$P_K = t^{-2} + t^{2} - 2$$

Theorem. $P_K(t)$ is a virtual knot invariant and

1) If K classical, then
$$P_K(t) = 0.$$

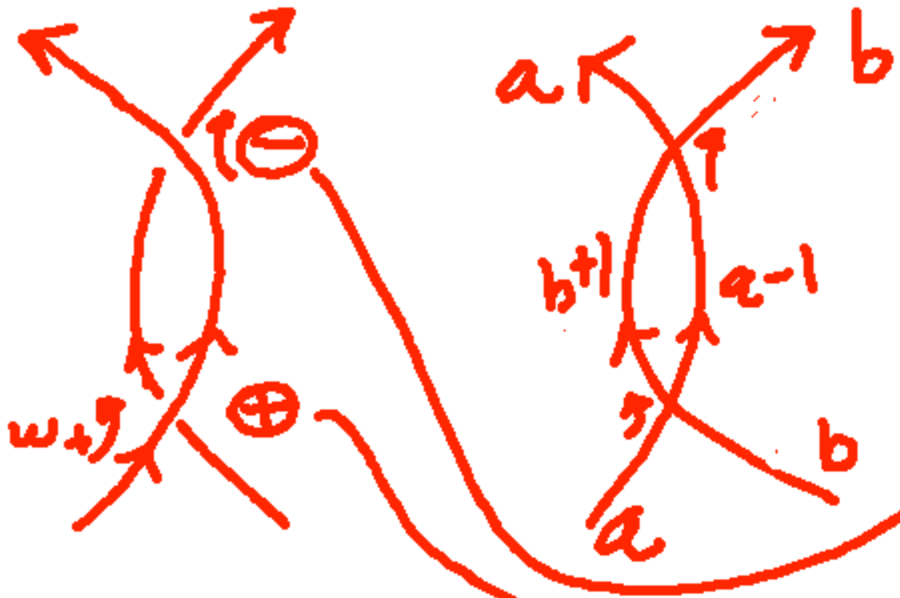
2) $P_{K^*}(t) = -P_K(t^{-1})$

K^* = mirror image of K (reverse all crossings)

3) $P_{\overleftarrow{K}}(t) = P_K(t^{-1})$

$\overleftarrow{K}$ = K with reversed orientation.

4) P_K is a concordance invariant. (next week)



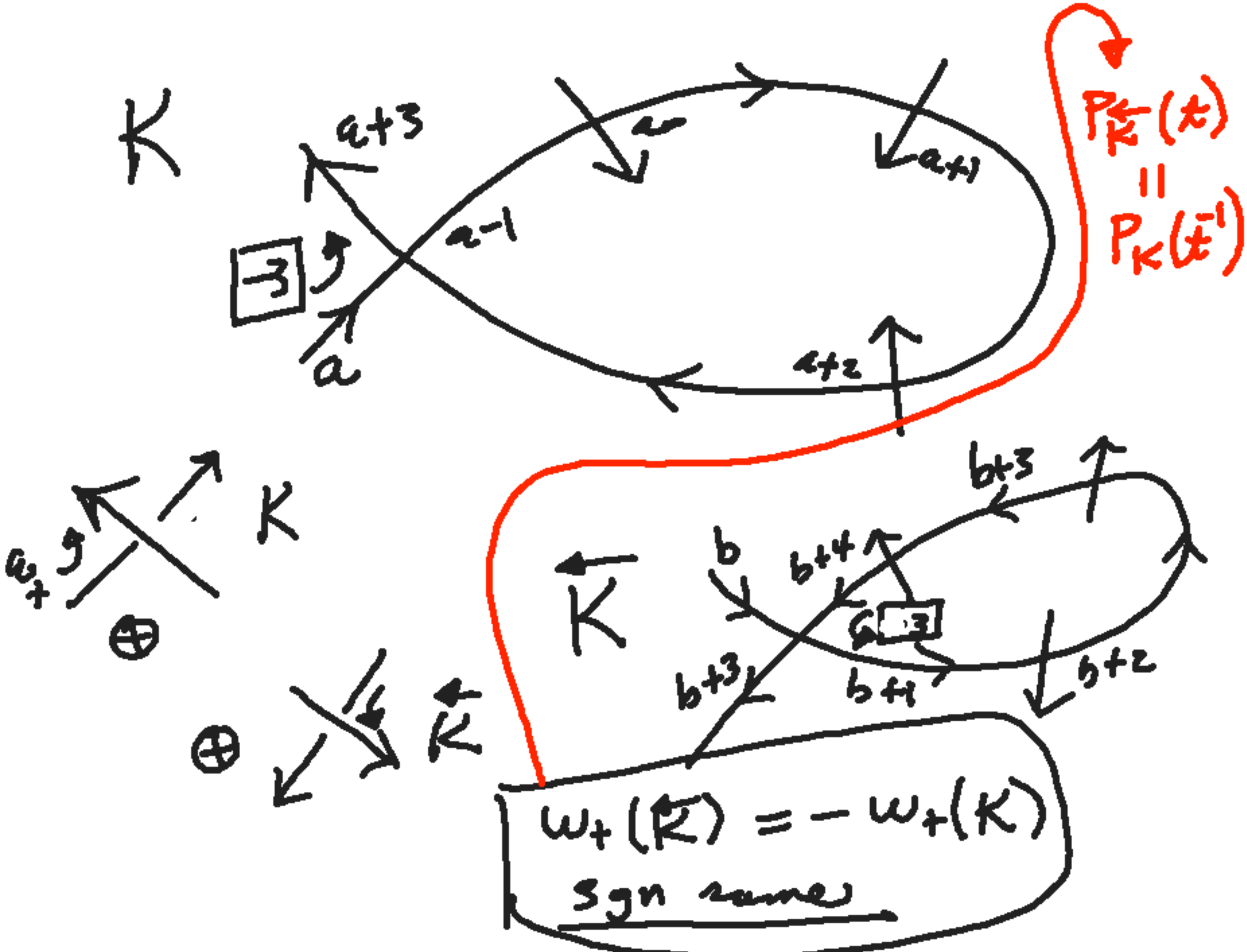
Exercise.

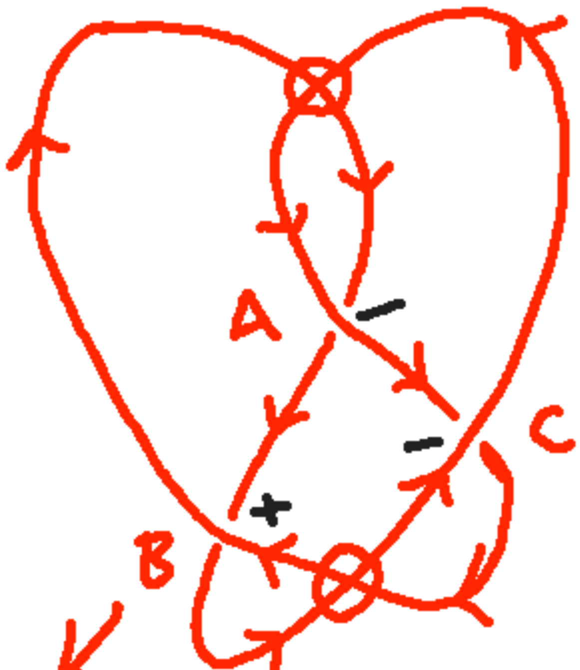
(a) (b) R3

$$+t^{a-b+1}$$

$$-t^{a-1-b}$$

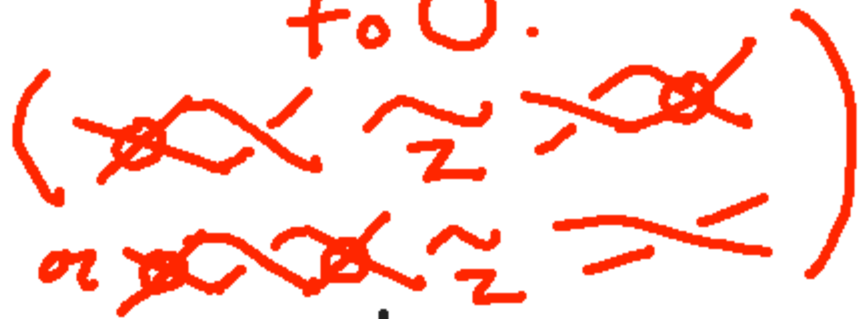
$$= 0$$


K
a+3
a
a+1
a-1
-3
a
a+2
$P_{\overleftarrow{K}}(t)$
=
$P_K(t^{-1})$
K
w_r
⊕
⊕
$\overleftarrow{K}$
$\overleftarrow{K}$
b
b+4
b+3
b+3
b+1
b+2
-3
$w_r(\overleftarrow{K}) = -w_r(K)$
sgn same



K

Exercise: K is Z-equiv to O.



$P_K = -t^{-2} + t - t^{-1} + 1$

$P_{\bar{K}} = -t^{-2} + t^{-1} - t + 1$

$P_{K^*} = t^{-2} - t^{-1} + t - 1$

	W_+	W_-
A	-2	2
B	1	-1
C	1	-1

$K \# \bar{K}$

$K \# K^*$

$K^* \# \bar{K}$