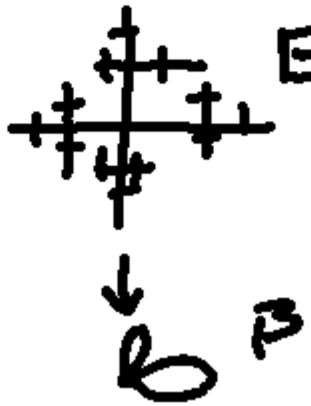
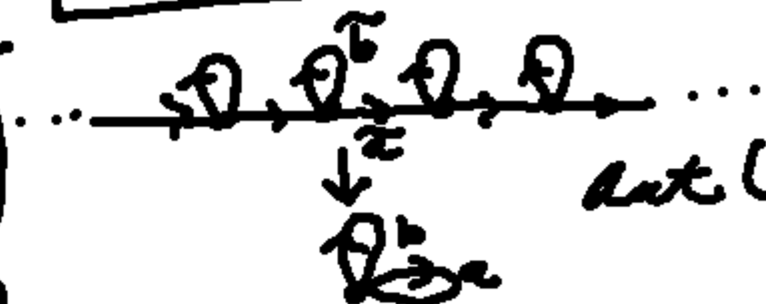
1. $\exists$ more remarks about covering spaces + alex polyn.



$\mathrm{Aut}(E/B) \cong (x, y \mid\ )$

$$\boxed{\widetilde{w\tau} = \tilde{w} + w\tilde{\tau}}$$

ex:



$\mathrm{Aut}(E'/B) \cong \mathbb{Z}$

$\pi_1(B) \longrightarrow \mathbb{Z}$

2. Virtual Knot Theory

a) Diagrammatic Defn.

b) Topological Interp.

Diagrams on a surface (usually oriented)

We can do knot theory on $T = S^1 \times S^1$.

$\longleftrightarrow$ embeddings of circles in $T \times [0,1]$.

(use) Reid Moves

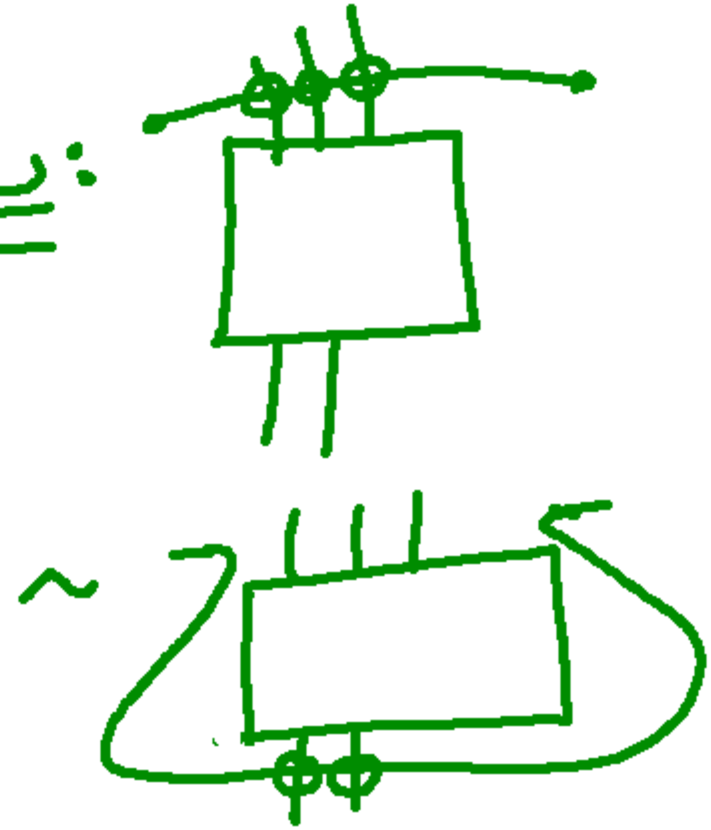
 virtual crossing

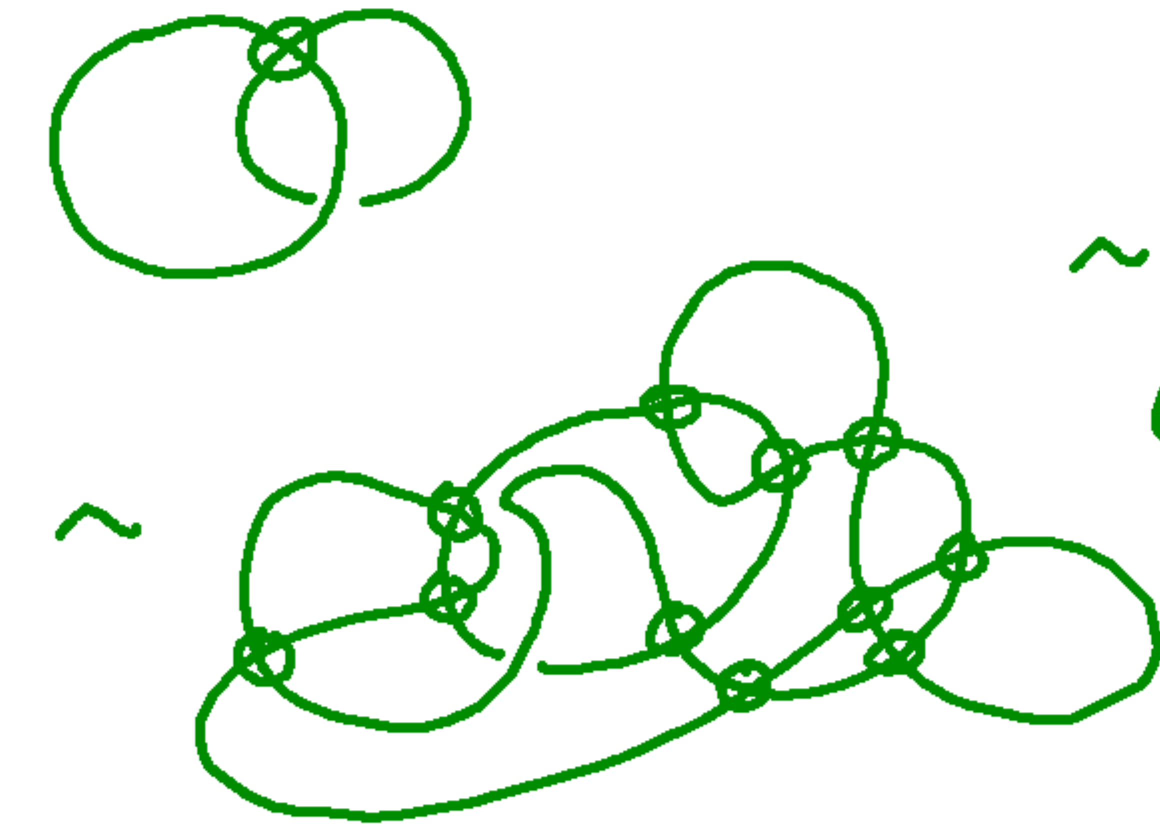
We will describe a planar diagrammatic theory that $\longleftrightarrow$ Stabilized Knots in Thickened Surfaces.

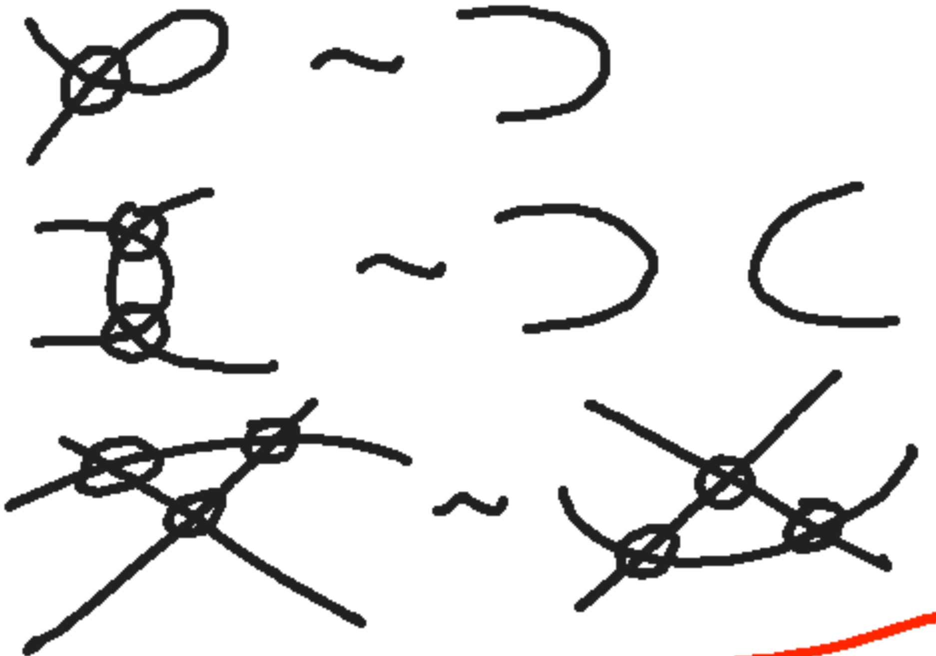
Add $\otimes$ virtual crossings.

Keep usual RM's.

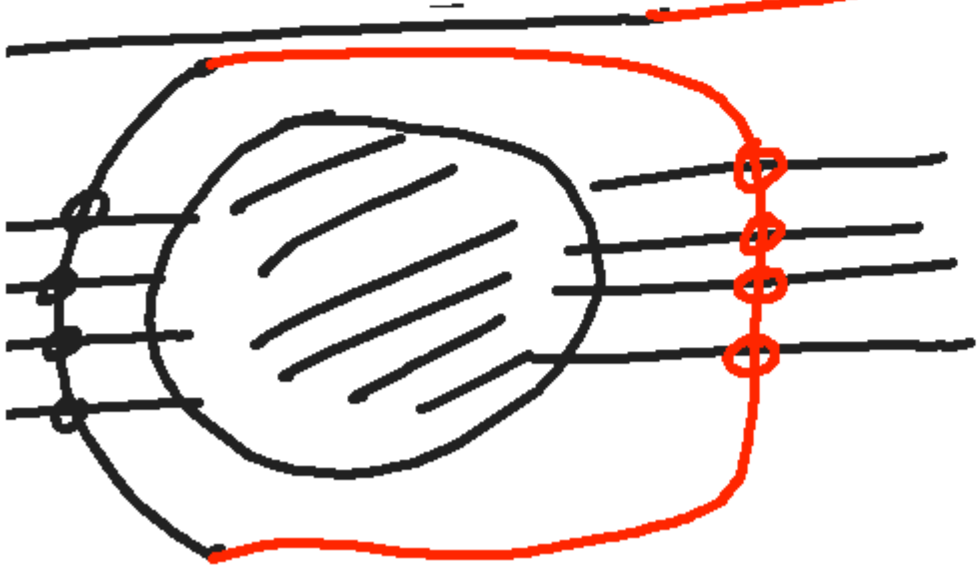
Add <u>Detour Move</u>:



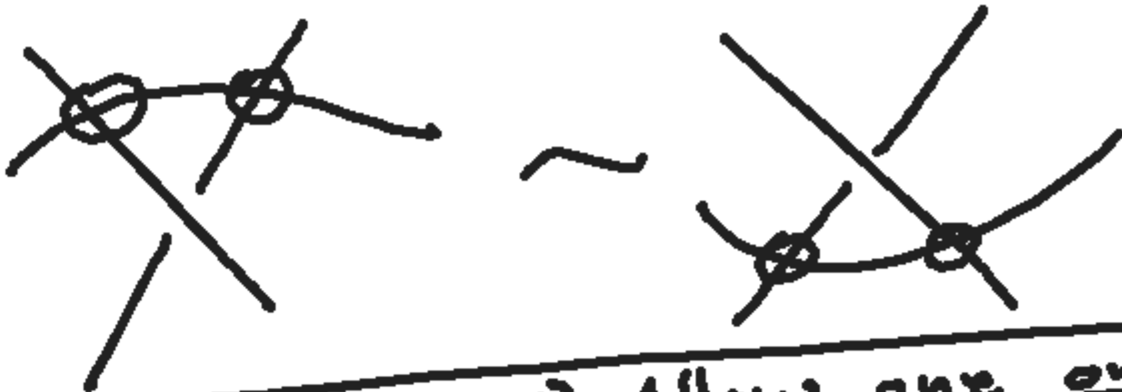




These generate all detours.

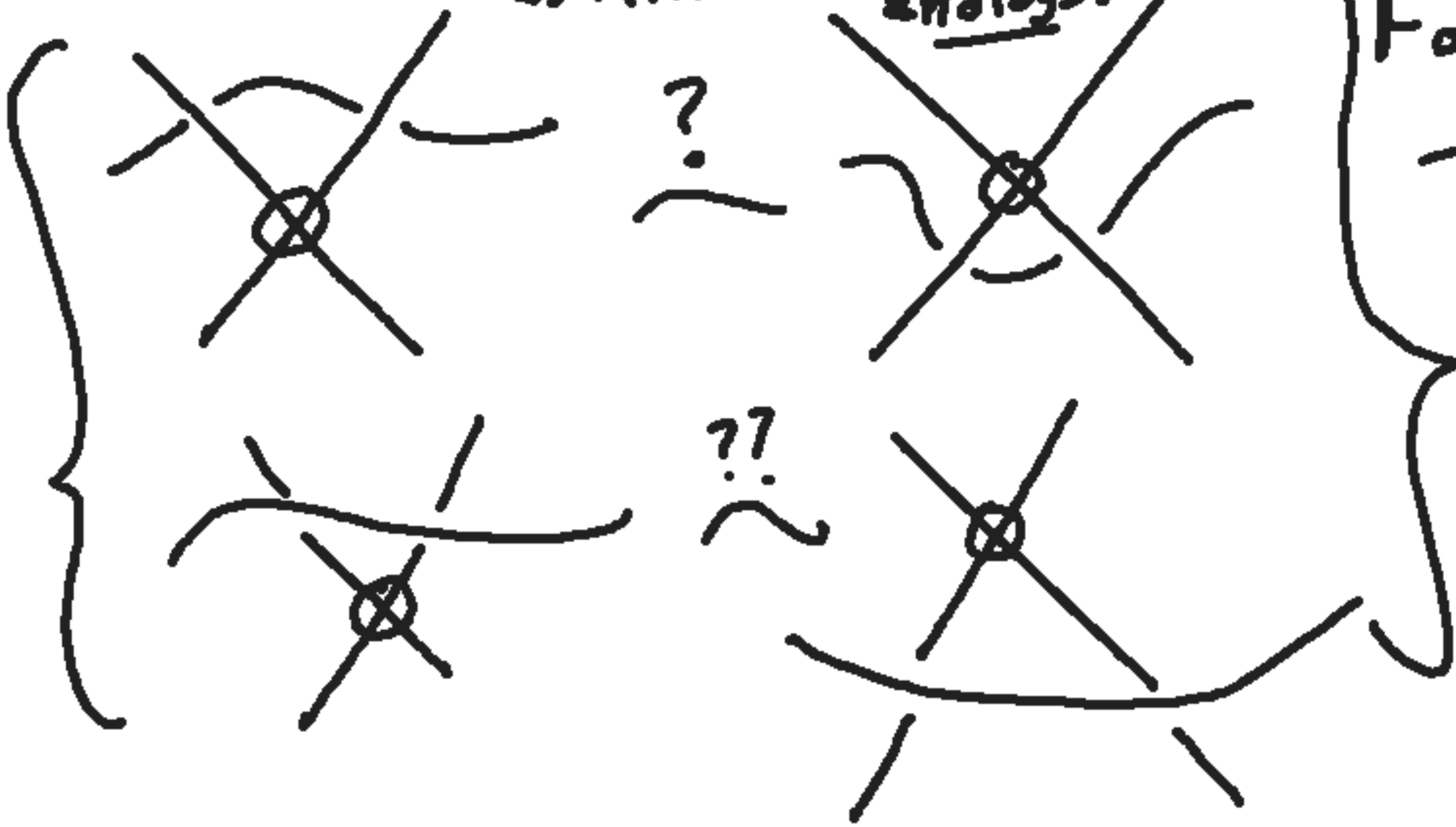


We shall have virtual braids.



1) Allow one only one gives a new theory: Welded Knots

2) Allow both, unknots all diags.



Forbidden!

U $lk(U) = 0$

link components A, B

$$lk(A,B) = \sum_{c \in Cr(A,B)} sgn(c)/2 .$$

L

$lk(L) = \frac{1}{2}$

$lk = 2$

Invar RM's ✓

Detour ✓

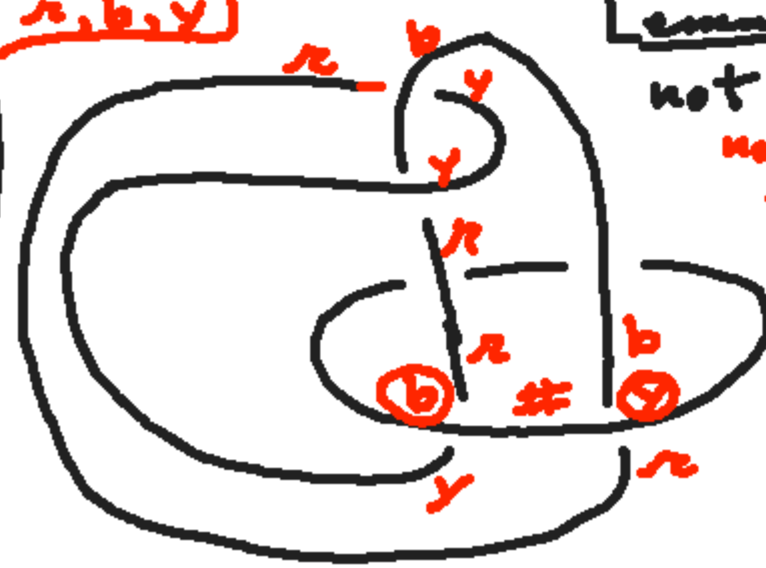
$\Rightarrow$ $lk(A,B)$ invar under all V moves.

L

$lk(L) = \phi$ but it is linked.

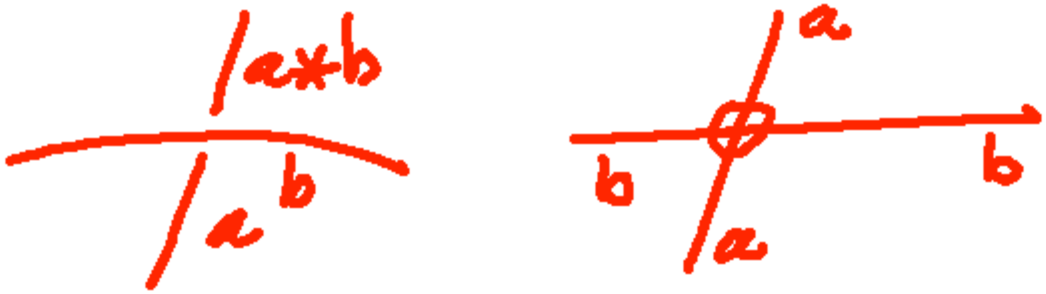
L not 3 col $\Rightarrow$ L linked

(r, b, y)

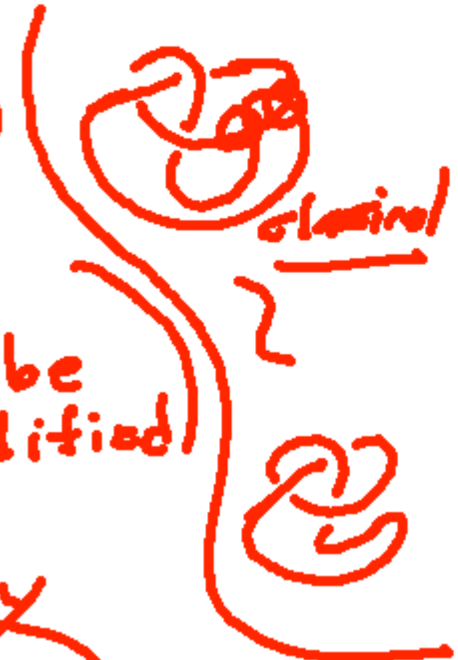


Lemma: L not 3 col. nontriv

Extend coloring (+ quandle) to virtuals via



this can be modified



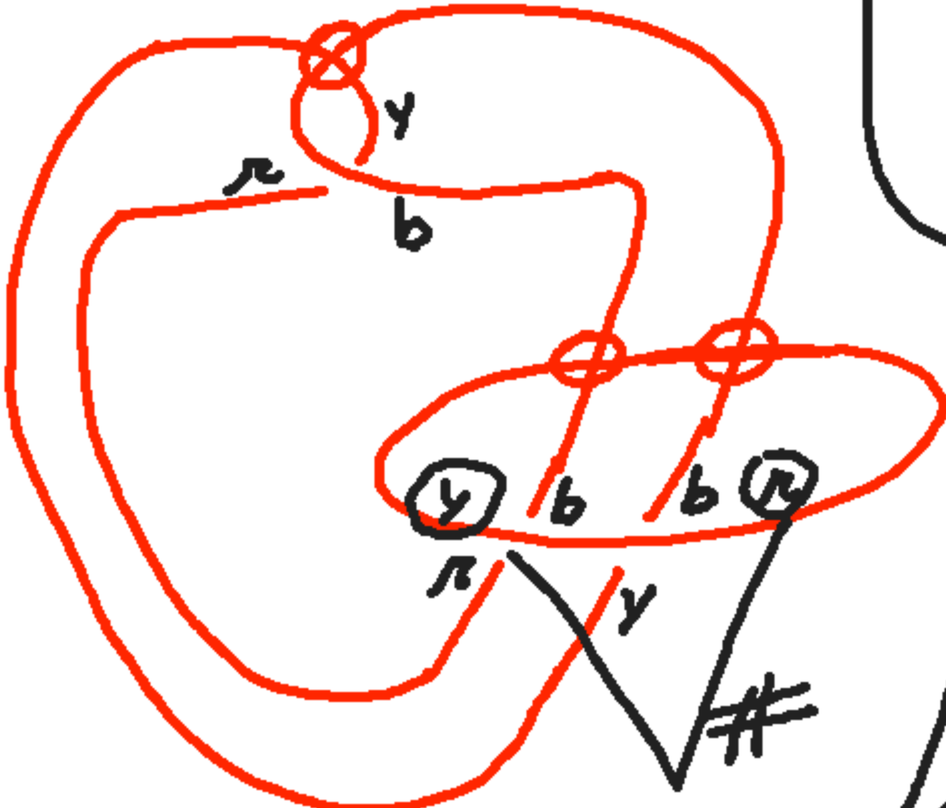
unicolor   unicolor

This 3 col ✓

∴ non-trivial.

But we have yet to prove it is not classical!

K is classical if $K \underset{\text{virtual moves}}{\sim} K'$ ← diag without $\otimes$.



$\langle K \rangle$ invar R2, R3, Detour

f_K invar R1, R2, R3, Detour

$\langle K \rangle$ extends to virtuals. all defns same & we take

$$\langle \text{loop} . K \rangle = \delta \langle K \rangle$$

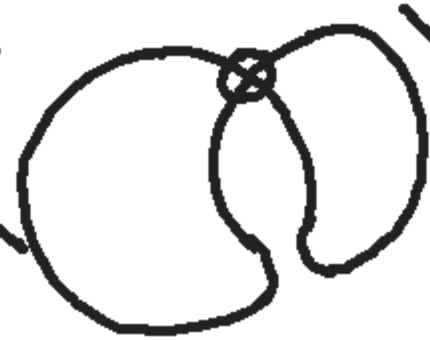
$$\delta = -A^2 - A^{-2}$$

loop can have $\otimes$.

$$\langle \text{loop} \rangle = 1$$

$$= 1.$$

$$f_K(A) = (-A^3)^{-wr(K)} \langle K \rangle$$

$\langle$  $\rangle = A \langle$  $\rangle$

L $\boxed{wr = 1}$

$= A + A^{-1}$ $\qquad + A^{-1} \langle$  $\rangle$

$\langle$  $\rangle = -A^{2} - A^{-2} = f_{OO}$

$\langle$  $\rangle = -A^{-3} \langle$  $\rangle$

$$f_L = (-A^3)^{-1}(A + A^{-1}) = -A^{-2} - A^{-4}$$

$\langle$  $\rangle = A \langle$  $\rangle + A^{-1} \langle$  $\rangle$

$$= A(A + A^{-1}) + A^{-1}(-A^{-3}) = A^{2} + 1 - A^{-4}$$

$\Big\langle$  $\Big\rangle = A \Big\langle$  $\Big\rangle + A^{-1} \Big\langle$  $\Big\rangle$

K

$$= A(A + A^{-1}) + A^{-1}(-A^{3}) = A^{2} + 1 - A^{-4}$$

$wr = +2$

$$f_K = (-A^3)^{-2}\left(A^{2} + 1 - A^{-4}\right)$$

$$f_K = A^{-4} + A^{-6} - A^{-10}$$

invar R1, R2, R3, Detour

$$f_{K^*} = A^{4} + A^{7} - A^{10}$$

$$\Rightarrow K \not\simeq K^*$$ (⊗ necc)

K is not classical!

Pf. all exponents in f_K, K classical are divisible by 4. $\boxed{\text{exercise!}}$

$= A$ 

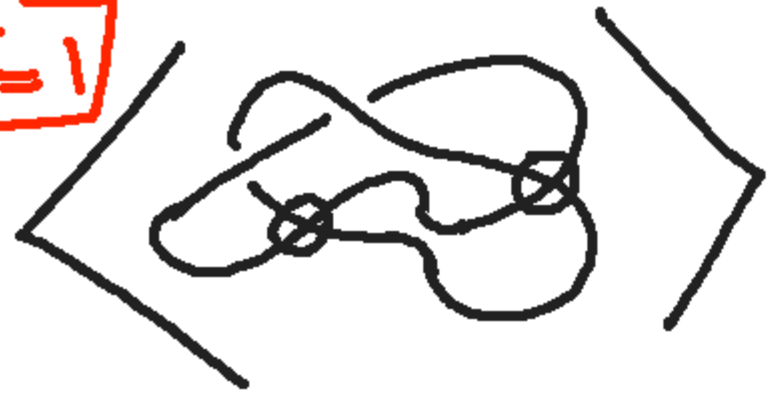
K

Nontrivial V-Knot with Jones Poly = 1

$\Longrightarrow \langle K \rangle = -A^3$

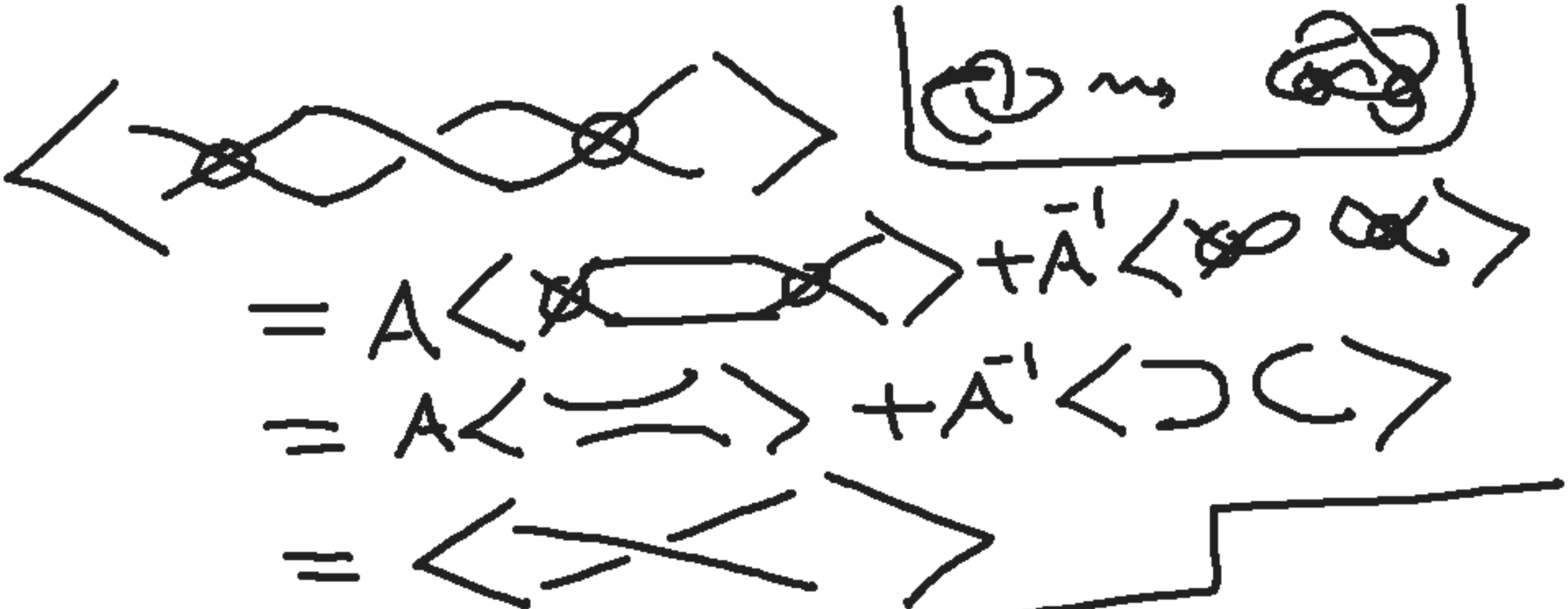
$\boxed{f_K = 1}$

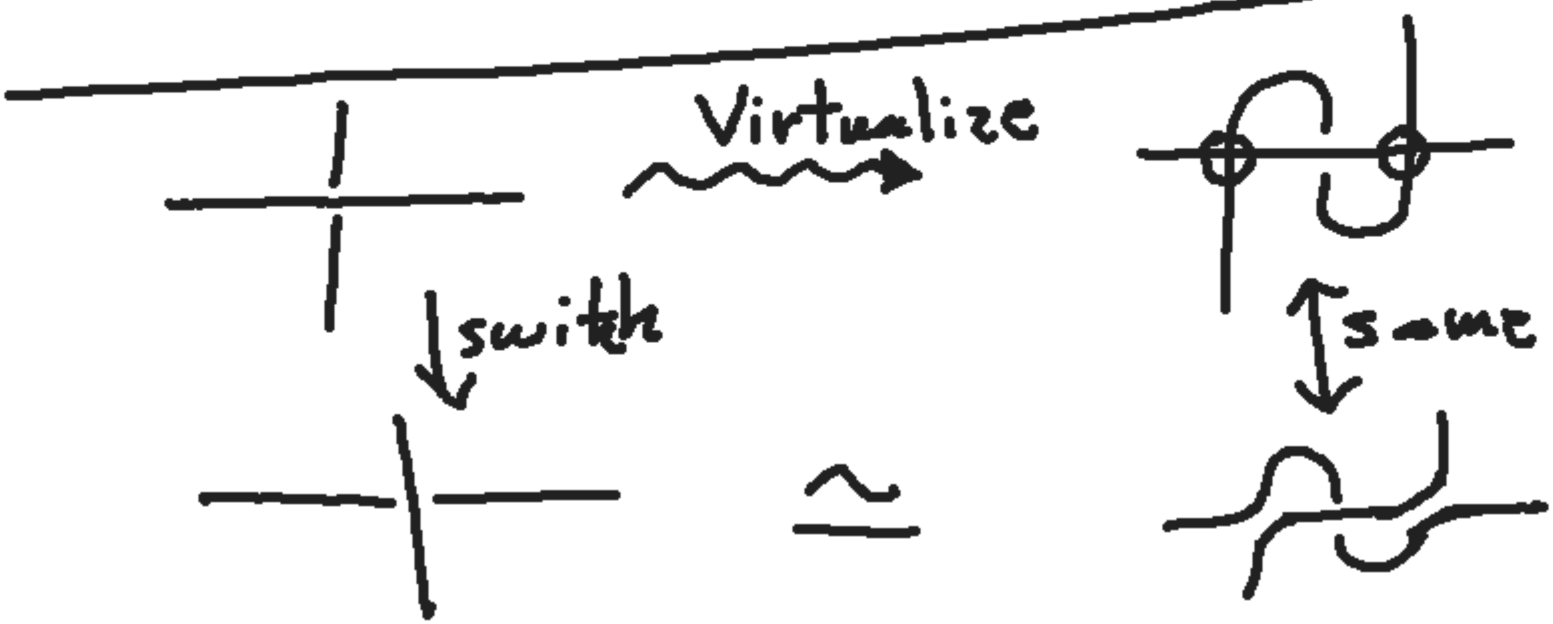
cd $\Rightarrow$ K nontrivial

$+ A^{-1}$ 

$= A$  $+ A^{-1}$ 

$=$  $=$  $= -A^3$

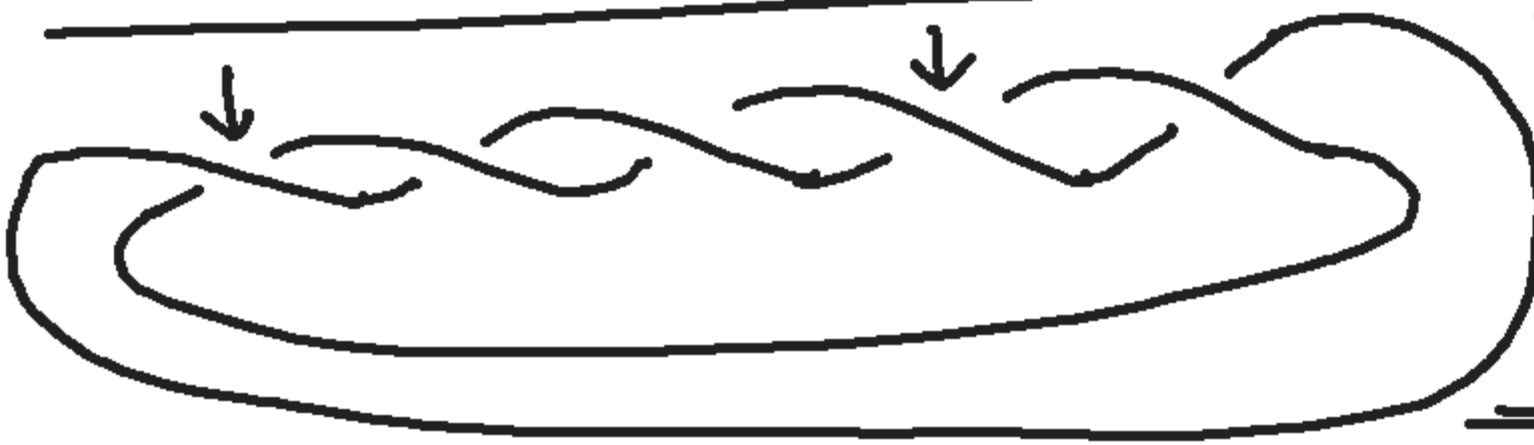

= A
+ A⁻¹
= A
+ A⁻¹
=


Virtualize
switch
same < >
≃

Take any classical diag K
choose a subset $S <$ Crossings(K)
+ virtualize them to get $\tilde{K}$.
But make sure that $K \rightsquigarrow$ Unknot
when you switch all of S.

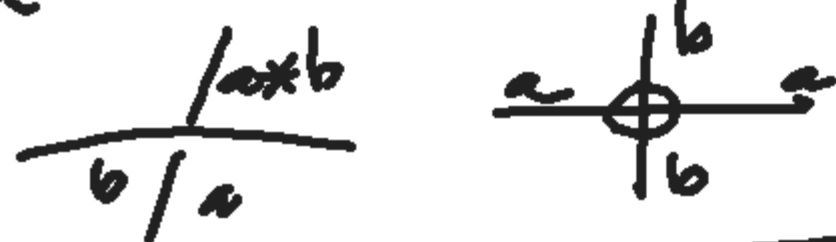
Then $f_{\tilde{K}} = 1$.

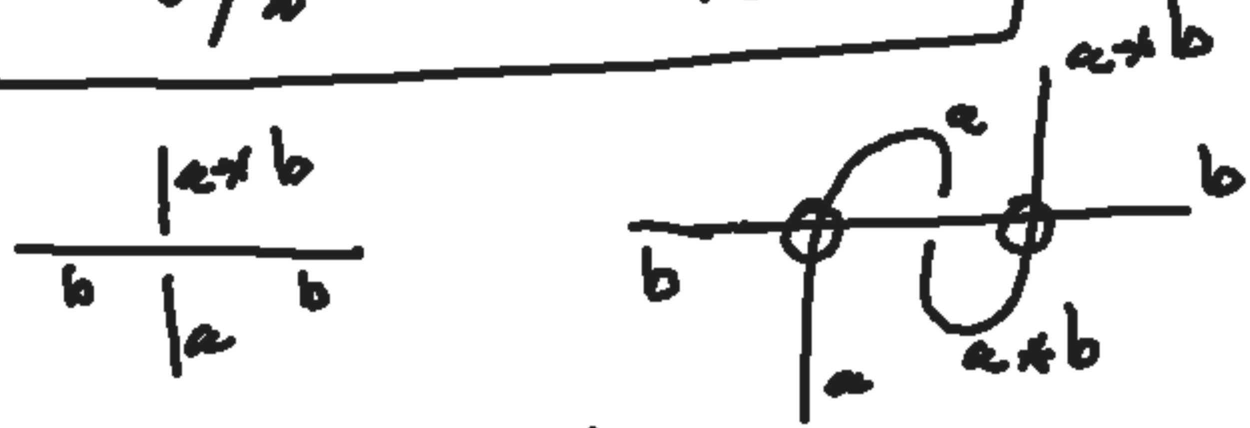
$\exists \infty$ by many $\tilde{K}$
$f_{\tilde{K}} = 1$.
+ non trivial
Pf next Time



Lemma. If $\tilde{K}$ is obtained from K by virtualization, then $Q(K) \cong Q(\tilde{K})$

Q = unoriented quandle.





no change

Fact. If K classical knot & knotted then $Q(K)$ non trivial