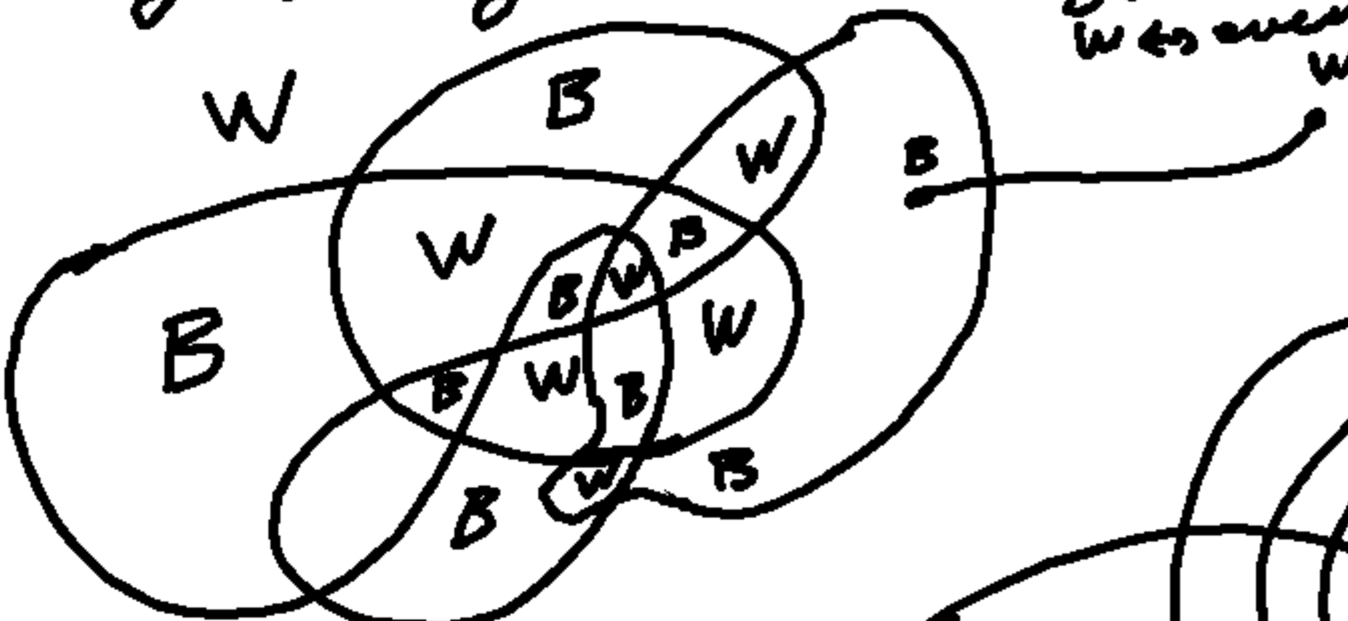
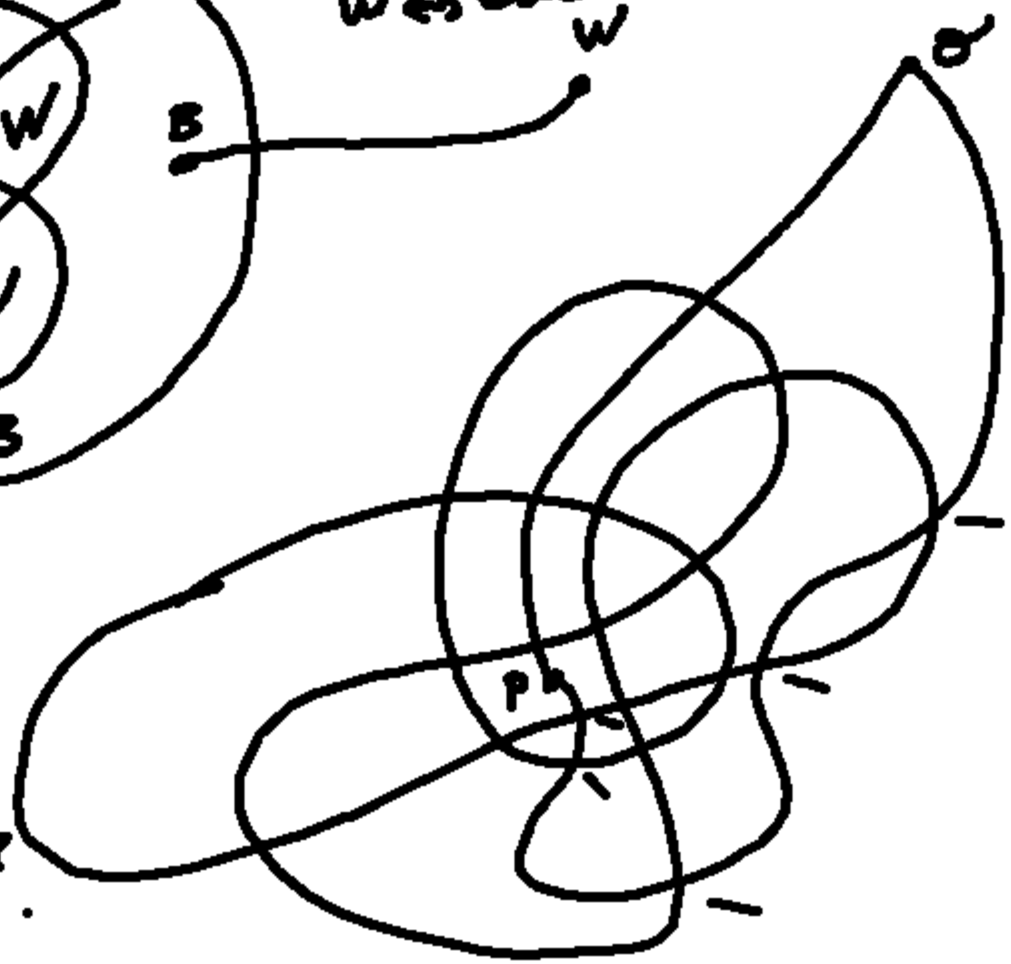
1. Show cannot make a PL knot with < 6 sticks. (next time!)

2. Prove $\exists$ checkerboard col of any diag.

B $\leftrightarrow$ odd
W $\leftrightarrow$ even



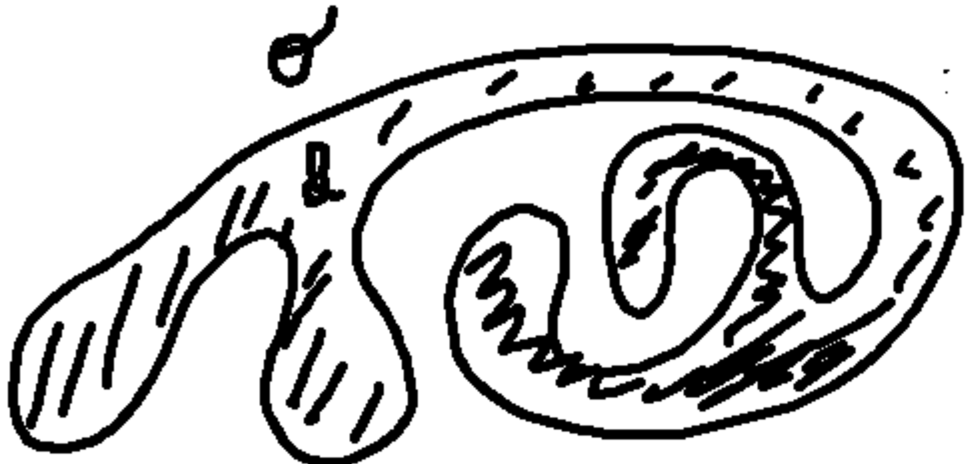


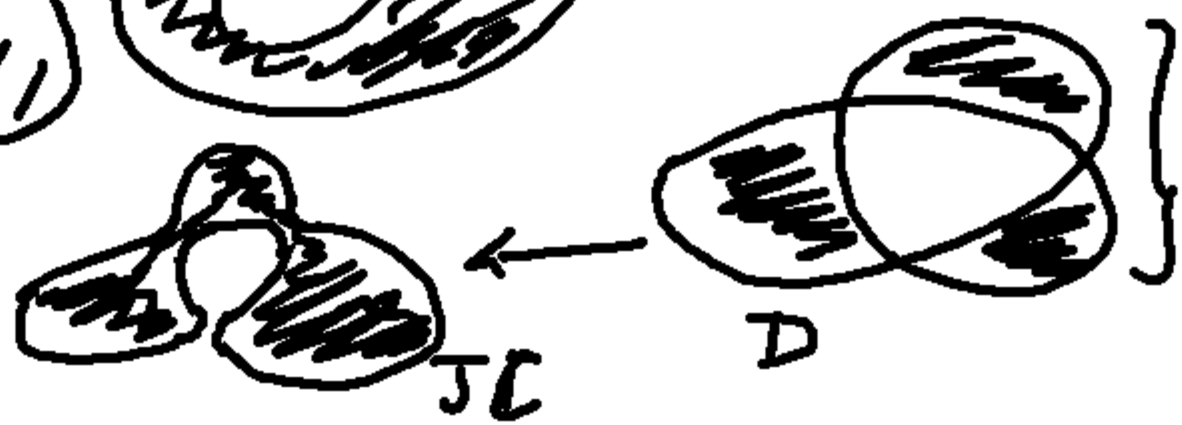
puzzle: why do all paths yield same parity?

Jordan Curve Theorem

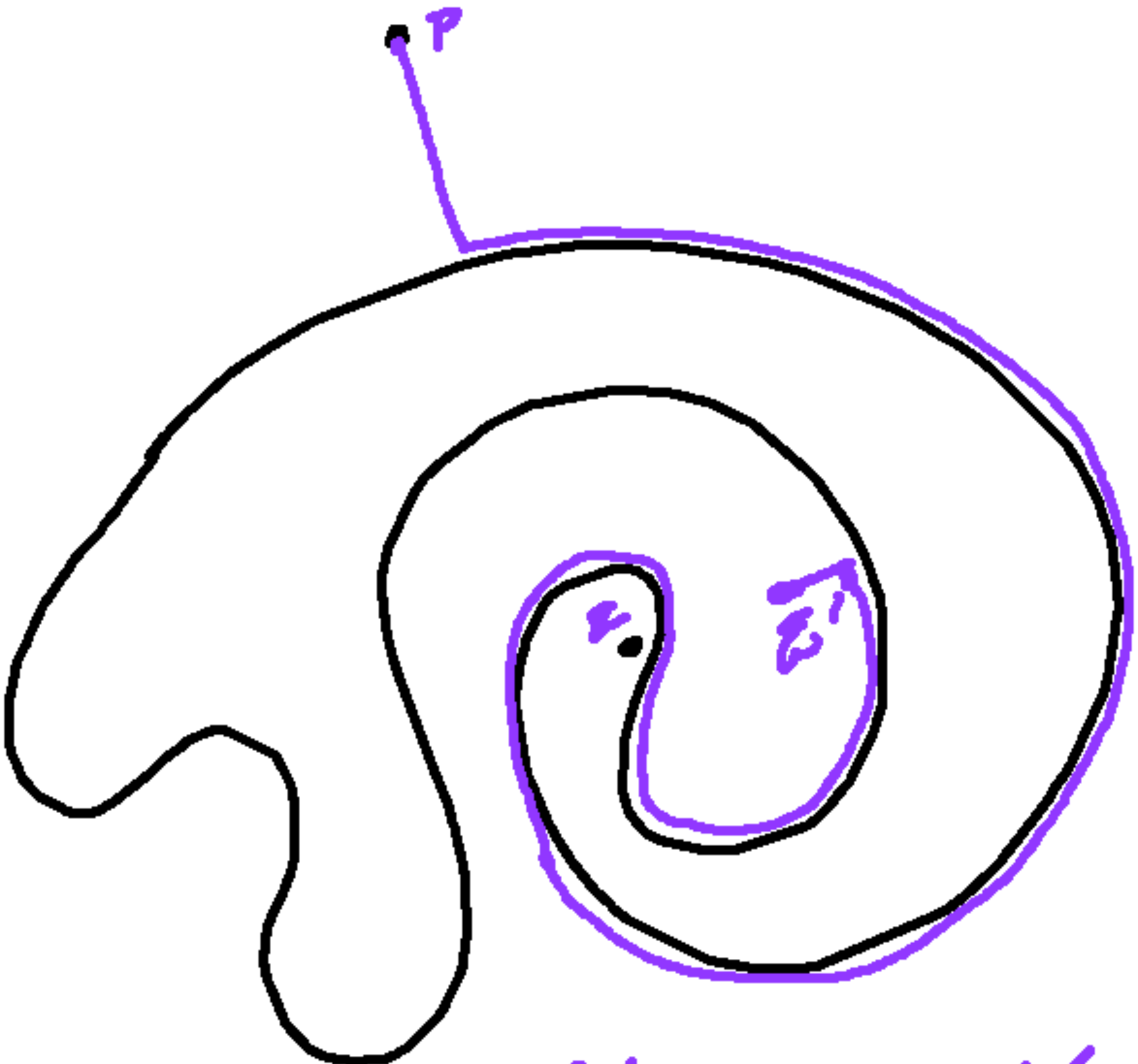
A simple closed curve in $\mathbb{R}^2$ divides it into two regions each homeom to a disk.

Convert a flat knot diag to a Jordan curve.





connected diagram



$\pi(p)$

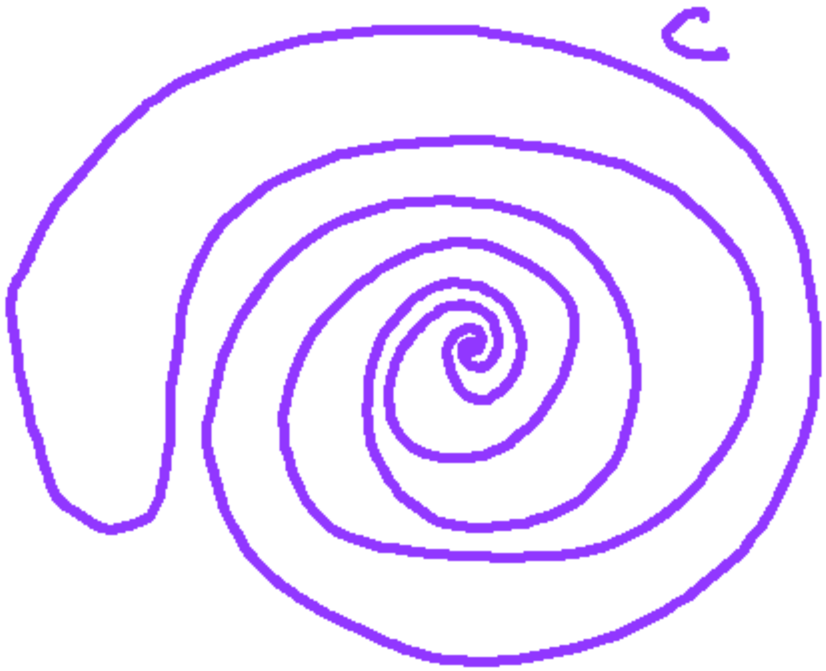
If parity(p) = parity(q) then $\exists$ a path from p to q not crossing the curve C.

$\pi(p) \neq \pi(q)$

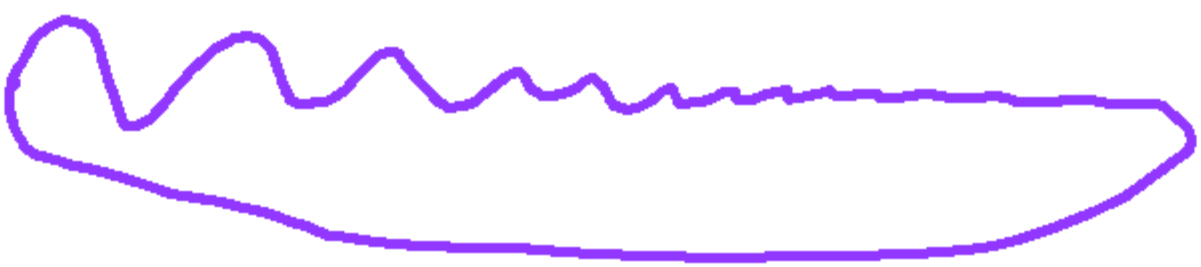
$\pi(p) = \pi(q')$

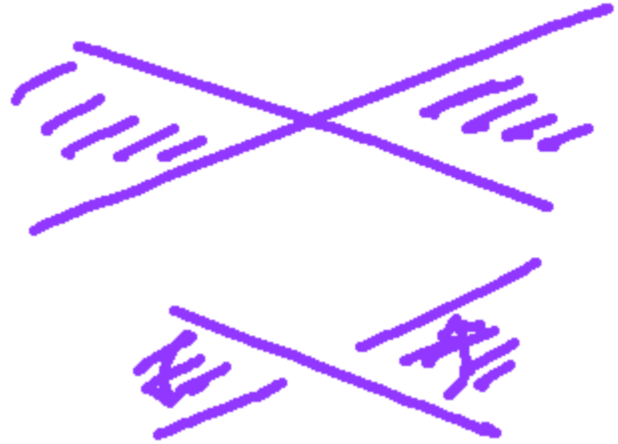
You can think about details.

If the curve has infinitely many wiggles, the problem is more difficult.



Here a st line would $\cap$ C in ∞ly many pts. So parity argument would not work there.





alter-nating

CB rule

$\Rightarrow$ alt

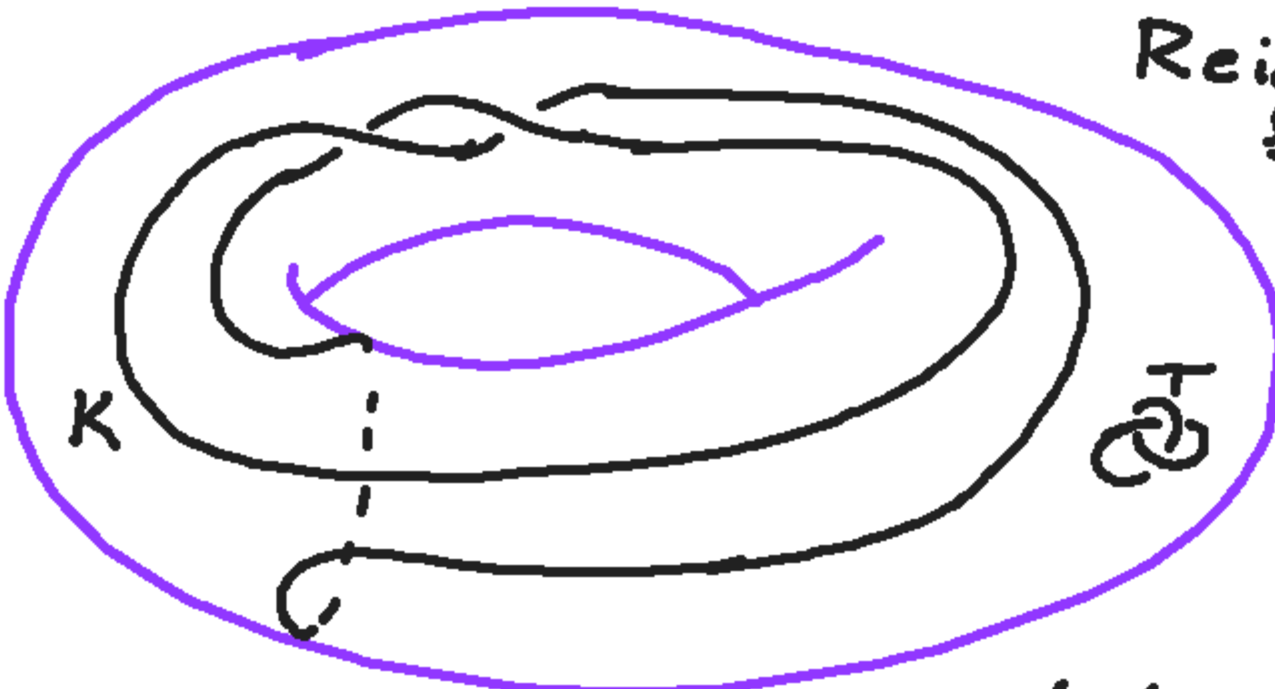
Torus - Two Curves

$\cong$ 

rectangle

Consider knot diagrams drawn on a higher genus surface. e.g. on a torus.

We can do Reid Moves for knot diags on the surface.



These can be regarded as

$S^1 \hookrightarrow T \times [0,1]$

What about knot diagrams on a Möbius Strip?

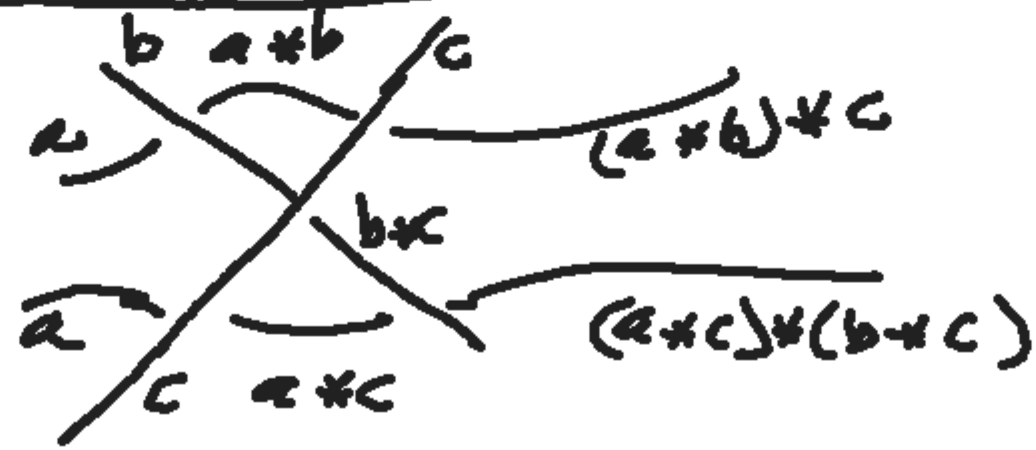
$\mathcal{M}$: • one edge
• one side

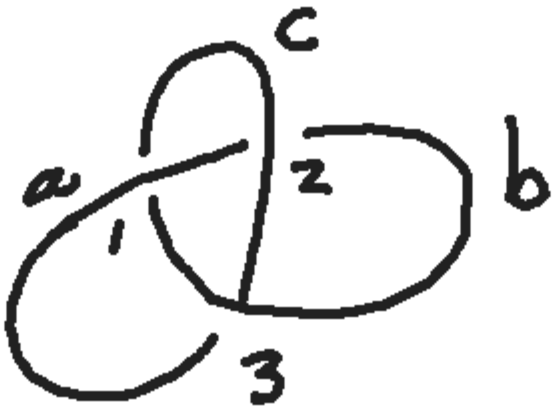
How will you do a diagrammatic knot theory?

Recall Unoriented Quandles

$\mathcal{A}$, binary op $*$

1. $a * a = a$
2. $(a * b) * b = a$
3. $(a * b) * c = (a * c) * (b * c)$





1. $b*a=c$, $c*a=b$

2. $a*c=b$, $b*c=a$

3. $c*b=a$, $a*b=c$

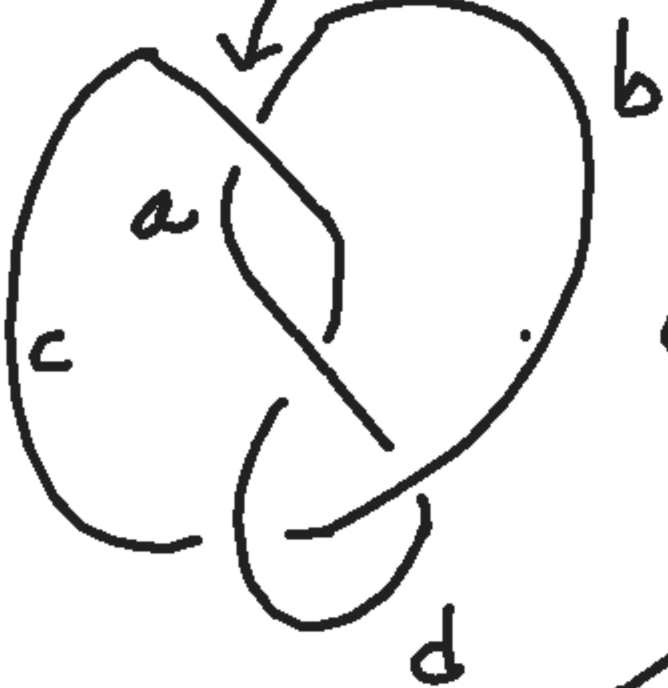
$$\left(\begin{aligned} b*a&=c \\ (b*a)*a &= c*a \\ b &= c*a \end{aligned} \right)$$

K: $Q(K) = \left(\underline{\underline{\text{gens}}} \,/\, \underline{\underline{\text{relations}}} \right)$

$*$	a	b	c
a	a	c	b
b	c	b	a
c	b	a	c

$$\begin{aligned} &((a*b)*c)*a \\ &= (c*c)*a \\ &= c*a \\ &= \underline{\underline{b}} \end{aligned}$$

$\underline{\underline{b*b=b}}$



$b = a * c$ arcs

$Q(K) = (g_1, \dots, g_n \mid$

$r_1, \dots, r_m)$

crossings

universal algebraic construction of a quandle with these generators and relations.

All words in gens. e.g. $(a*(b*c))*(d*b)$

modulo equiv rln gen by 1) relations. 2) quandle axioms.

Theorem. If $K \underset{RM}{\sim} K'$

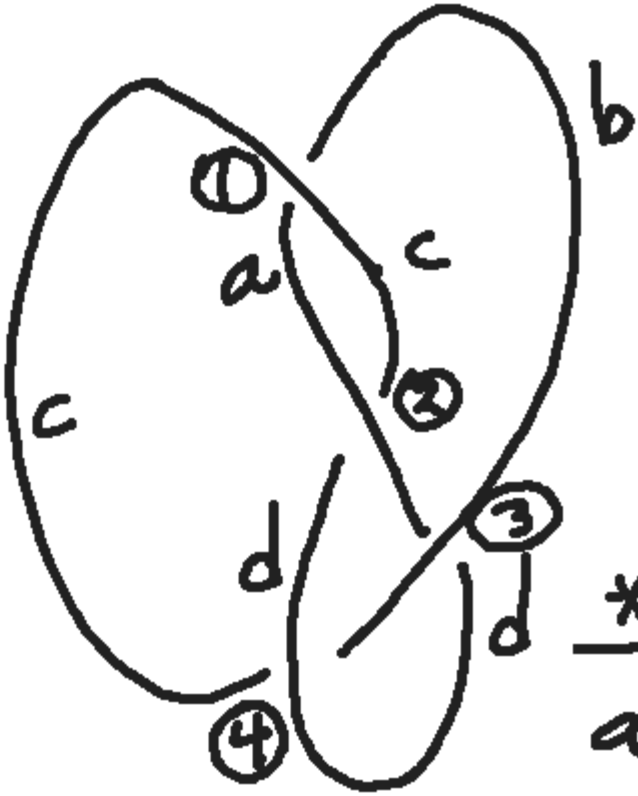
then $Q(K) \cong Q(K')$.

isomorphic

$Q($  $) =$

*	a	b	c
a	a	c	b
b	c	b	a
c	b	a	c

$Q($  $) = ?$



1. $a*c=b$, $b*c=a$
2. $d*a=c$, $c*a=d$
3. $d*b=a$, $a*b=d$
4. $b*d=c$, $c*d=b$

$*$	a	b	c	d	e
a	a	d	b	e	c
b	e	b	a	c	d
c	d	e	c	b	a
d	c	a	e	d	b
e	b	c	d	a	e

$$(ab)e = de = b$$

$$(ae)(be) = cd = b \checkmark$$

Check that this mult table satis Quandle axioms

$$
\begin{array}{c|cccccc}
 & a & b & c & d & e & \cdots \\
\hline
\varphi & r & s & t & u & v & \cdots \\
 & a*a & a*b & \cdots & & & \\
\end{array}
$$

all group elements.

$\varphi * (a, b, c, \ldots)$

Question! $\longrightarrow$ a perm of $(a, b, c, \ldots)$

$\boxed{\text{Show: Both rows + cols are permutations}}$

$(a, b, c, \ldots) * y$

$F_y : a \longrightarrow a$

$F_y(a) = a * y$

$F_y = F_y^{-1}$ $\therefore$ 1-1 onto

UL_2

$Q(UL_2)$

$= (a, b \mid \quad)$

$a^2 = a, \; b^2 = b$

$ab, \; ba$

$(((ab)a)b)a$

$\cdots$

H

$ab = a$
$ba = b$

$Q(H) = (a, b \mid ab = a, \; ba = b)$

$(x * y) * z = (x * z) * (y * z)$

$((x * z) * y) * z$
$= ((x * z) * z) * (y * z) = x * (y * z)$

Making linear quandles

$$a * b = ra + sb$$

r, s elts of $\mathbb{Z}$ or $\mathbb{Z}_N$

$+ a, b \in$ some module

e.g. just use $\mathbb{Z}$ or $\mathbb{Z}_N$ some N

$a * a = ra + sa$
$= (r+s)a$

$\boxed{\text{assume } r+s=1}$

$$(a*b)*b = r(ra+sb) + sb$$
$$= r^2 a + s(r+1)b$$

Want $\boxed{r^2=1}$ and $s(r+1) = 0$

so $r=-1$ $r+s=1 \Rightarrow s=2$

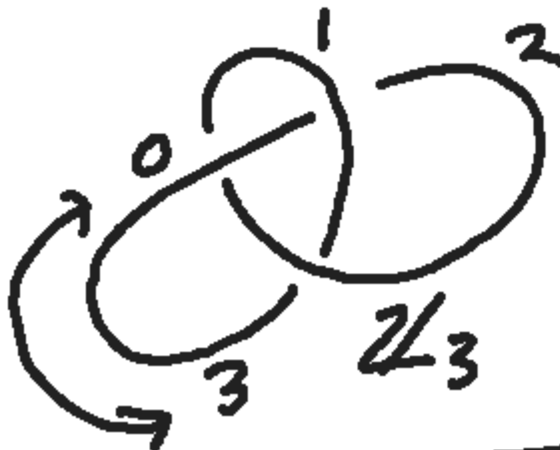
$\boxed{a*b = -a+2b}$

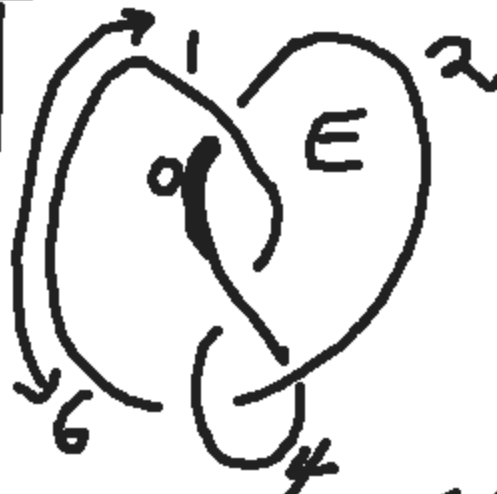
(so $s=0 \Rightarrow r=1$ not interesting)

Fox Coloring

$$a*b = 2b - a$$

Check $(a*b)*c = (a*c)*(b*c)$





$\exists$ other diags of E that demand all 5 cols.

$\mathbb{Z}_5$

$\{0, 1, 2, 3, 4\}$

Take a knot in knot table + find out as much as you can about its quandle.

Show mult table for this $\mathbb{Z}_5$ quandle is the one we found by "sudoku".

- can prove can't use < 4 colors on any non-trivial $\mathbb{Z}_5$ col of a knot.

Suppose K can be colored in $\mathbb{Z}_N$.

Let $\text{mincol}_N(K)$ = least number of colors coloring K over all diagrams $\underset{RM}{\sim} K$.

This is an elusive invariant of the knot K.

- virtual knots
- orient quandle of Alexander poly.
- bracket + Jones poly

Return to $Q(K)$. Use

$QF(K)$ = quandle using module$/\mathbb{Z}$ or $\mathbb{Z}_N$ with $a * b = 2b - a$

$QF(O) = (a|\ ) \cong \mathbb{Z}$ with $a * b = 2b - a$.

$QF(OO) = (a, b|\ ) \cong$? gen by a, b elts of a module over $\mathbb{Z}$.

so $QF(OO) \cong \mathbb{Z} \oplus \mathbb{Z} = \{ra + sb\}$

QF():

$a = a * b = 2b - a$

$b = b * a = 2a - b$

So $2(b - a) = 0$. $\therefore / \mathbb{Z}_2$.

$\mathbb{Z}_2$

So QF() finite
while QF(O O) infinite.

<u>Exercise</u>. Let G be a group written mult. product gh.

Define $a * b = b a^{-1} b$.

Show that this gives G structure of an unoriented quandle.