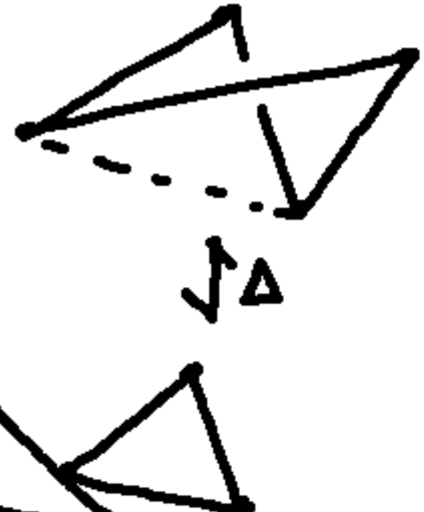
Recall: PL knots + links $\subset \mathbb{R}^3 \subset S^3$



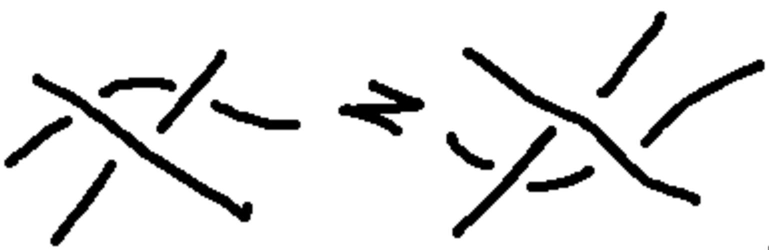
- Reid Alex Briggs Δ-moves.
- By projection get that diagrams + R-moves suffice for ambient isotopy

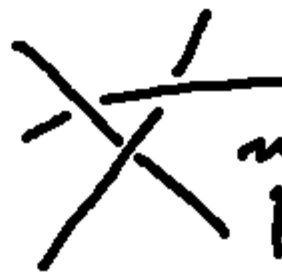
R-moves

0. 

I. 

II. 

III. 

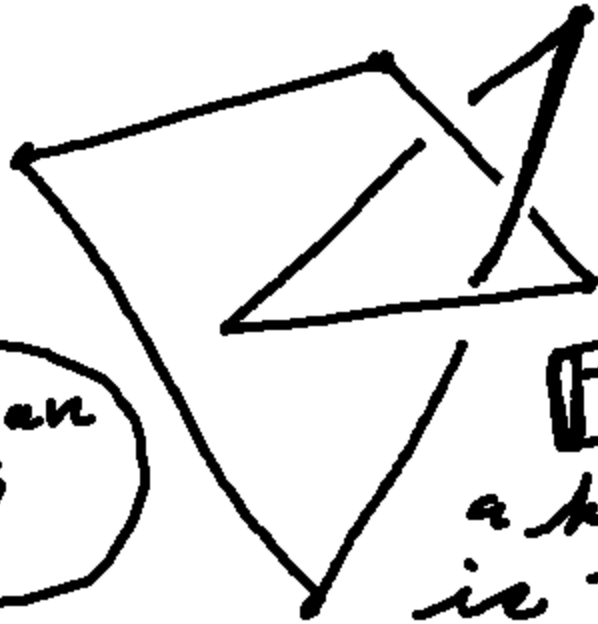


no R3 move here

There is a doggybox link.

Exercise. 6 edges are needed to make a knot.

T



$\boxed{\text{5 not possible}}$

we mean in $\mathbb{R}^3$

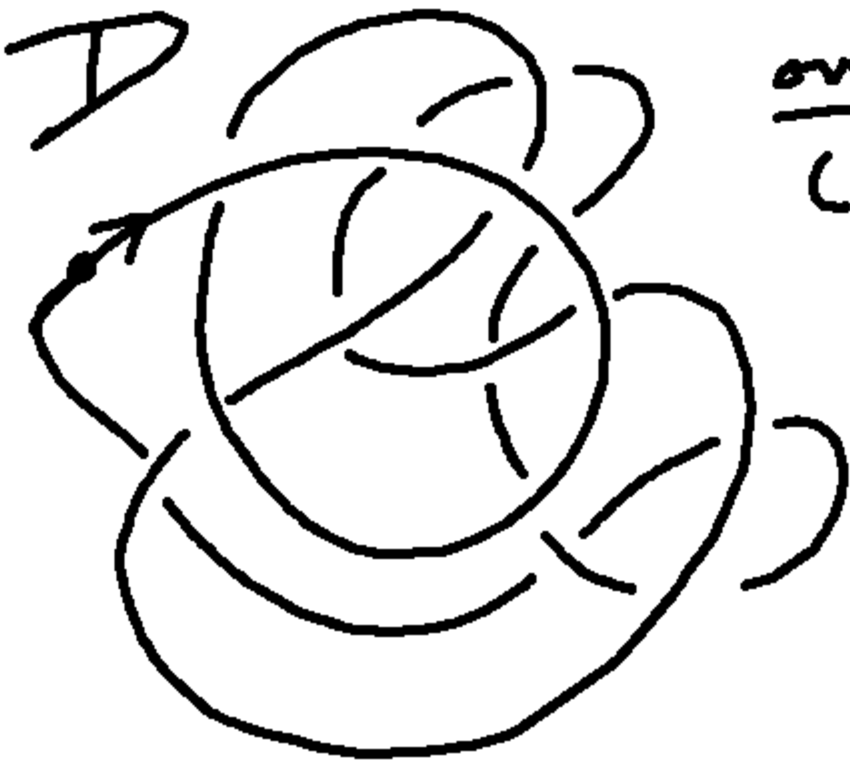
Problem: Given a knot K what is the least # of sticks needed to make K?

st line diags do not always realize as straight sticks in $\mathbb{R}^3$.

E

e.g.

D



overcross first

(descending diagram)

It is unknotted

RThm. $D \underset{RM's}{\sim} O$

D

$\boxed{\text{overcross first}}$

(descending diagram)

It is unknotted

$v=3$
$e=6$
$f=5$
$v-e+f=2$ ✓

RThm. $D \underset{RM's}{\sim} O$

Exercise. If a conn. flat diagram has no  and no , then it will have some .

Hint: conn. plane graph, $v-e+f=2$

colorable
with 2
colors
B, W.

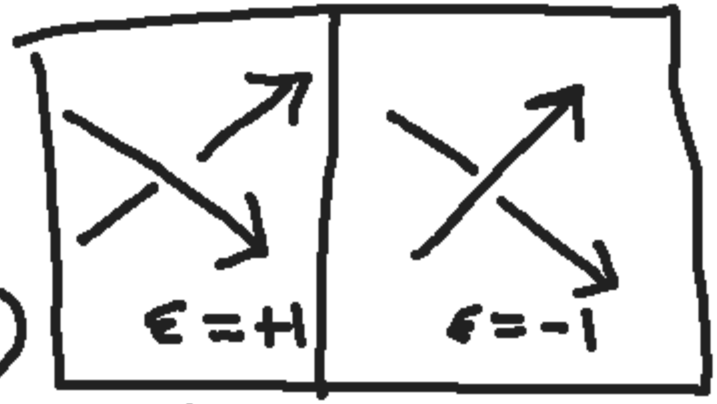
Every flat
diag (4 reg plane
graph) can
be 2 colored.

counter clock turn
of over cross line
sweep black regions.

⇒ resulting weave is alternating

orientation

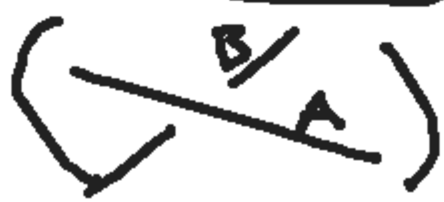
ex:  $\underset{RM}{\sim}$ 



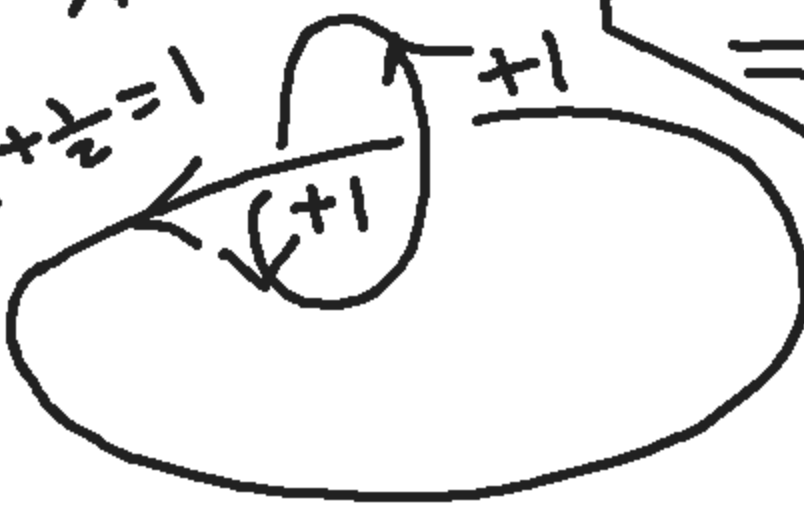
sign convention

$lk(A,B) = 1$

$$lk(A,B) = \sum_{c \in Cr(A,B)} \varepsilon(c)/2 .$$



here
$lk = \frac{1}{2} + \frac{1}{2} = 1$



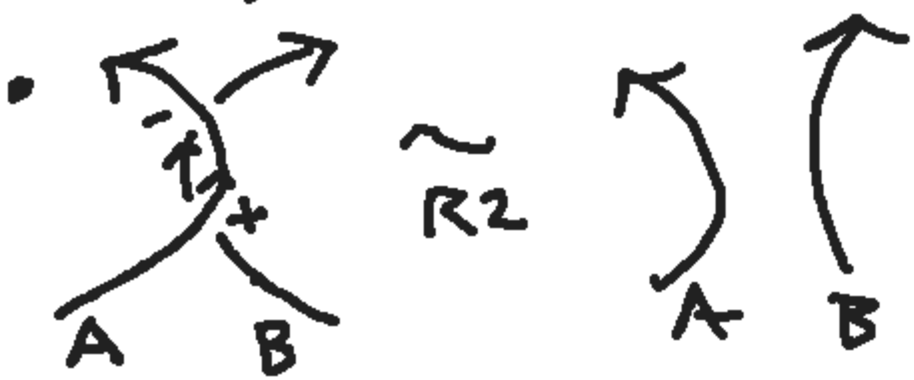
$L = A \cup B$ oriented

$$lk(L) = lk(A,B) = \frac{1}{2} \sum_{c \in Cr(A,B)} \varepsilon(c)$$

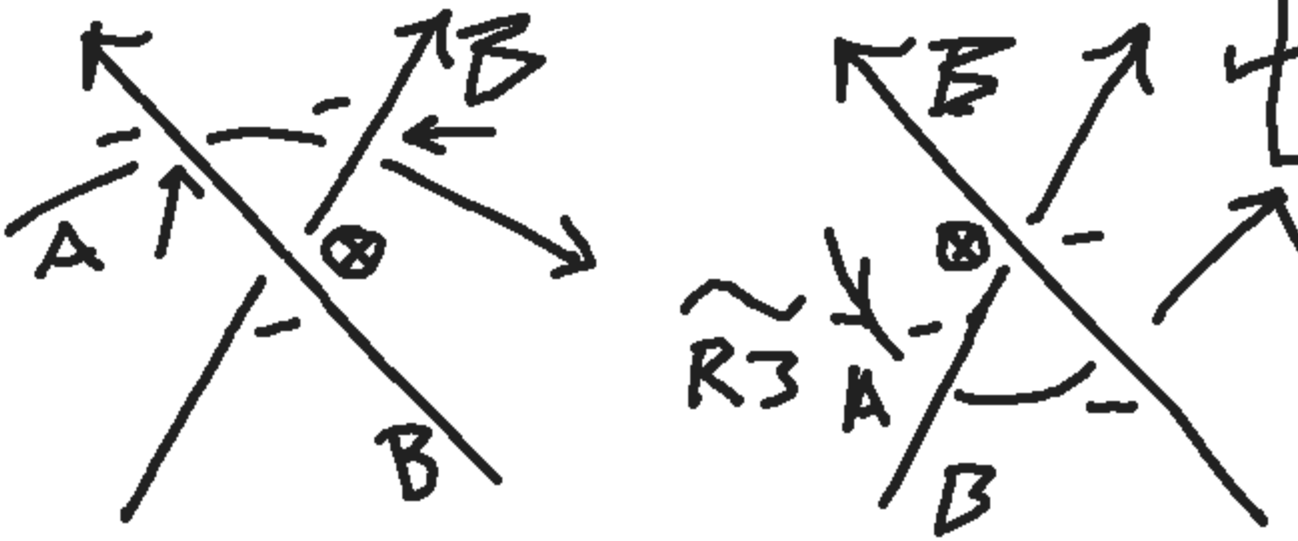
$\Longrightarrow lk(L)$ unchanged by RM's.

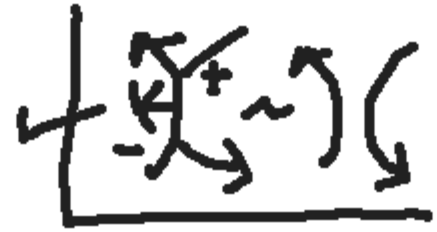
Proof: 

no contribution ($c \notin Cr(A,B)$)



$-\frac{1}{2} + \frac{1}{2} = 0$ ✓





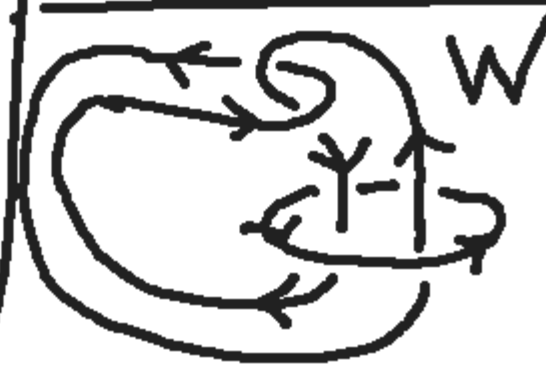
sum of $\pm$ does not change.
you can check other cases. //

$lk = 1$

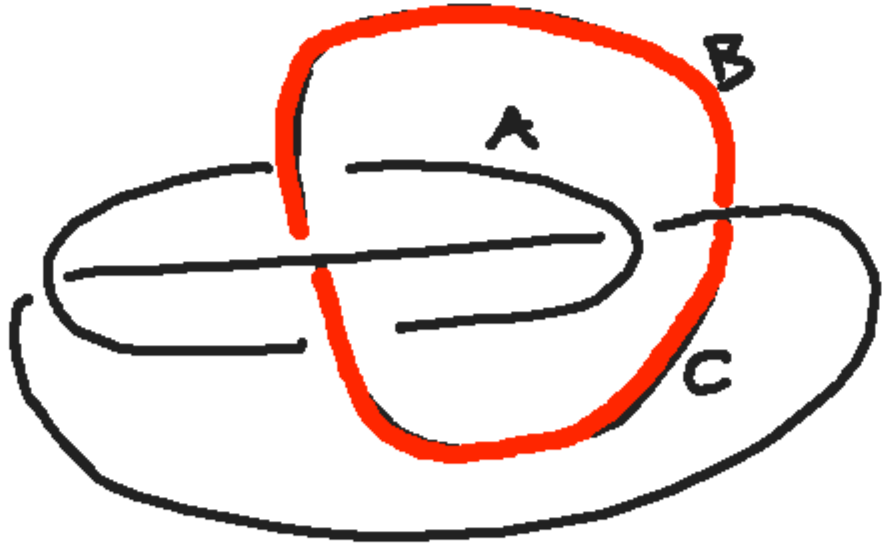
$lk = 2$

, . . .

$\exists$ lks with $lk\# = 0$ + still linked.



Borommean Rings



$lk(A,B) = 0$
$lk(A,C) = 0$
$lk(B,C) = 0$

Remove any one of A, B, C

then the link falls apart.

A over C over B over

3 color algebra

members r, b, g distinct

$*$	r	b	g
r	r	g	b
b	g	b	r
g	b	r	g

$r^2 = r$
$b^2 = b$
$g^2 = g$
$rb = g$ etc

not assoc: $(rb)g = gg = g$
$r(bg) = rr = r$

Properties

1. $xx = x$
2. $(xy)y = x$
3. $(xy)z = (xz)(yz)$

$(rg)(bg) = br = g$ ✓

3 col. algebra

members r, b, g distinct

$*$	r	b	g
r	r	g	b
b	g	b	r
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$r^2 = r$
$b^2 = b$
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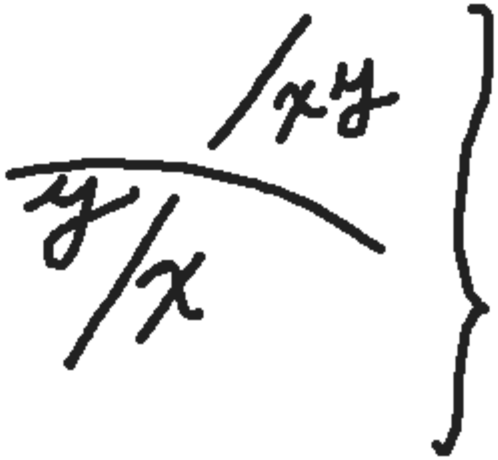
not assoc: $(rb)g = gg = g$
$r(bg) = rr = r$

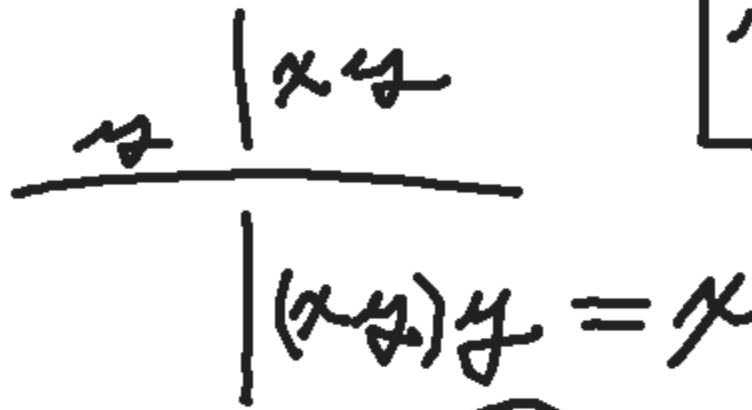
unoriented quandle

Properties

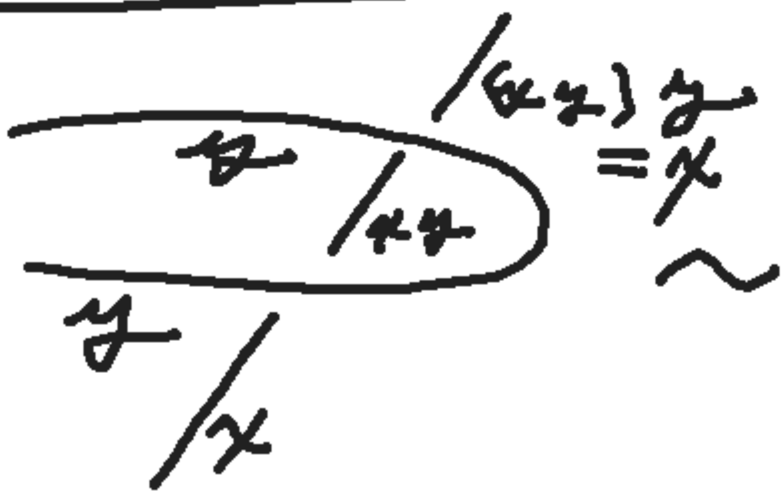
1. $xx = x$
2. $(xy)y = x$
3. $(xy)z = (xz)(yz)$

$(rg)(bg) = br = g$ ✓

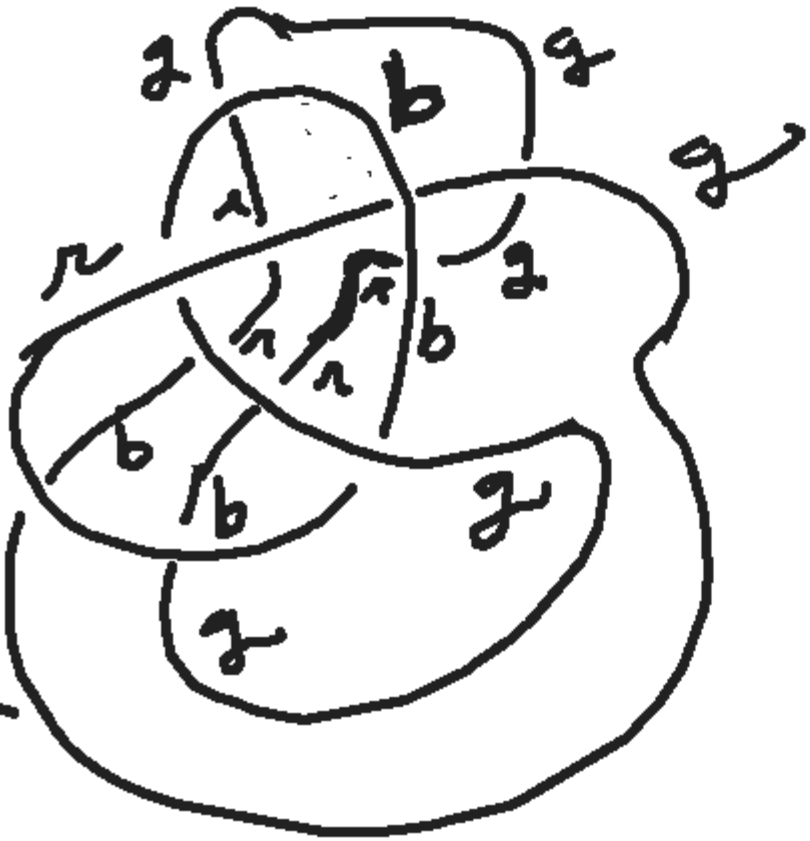


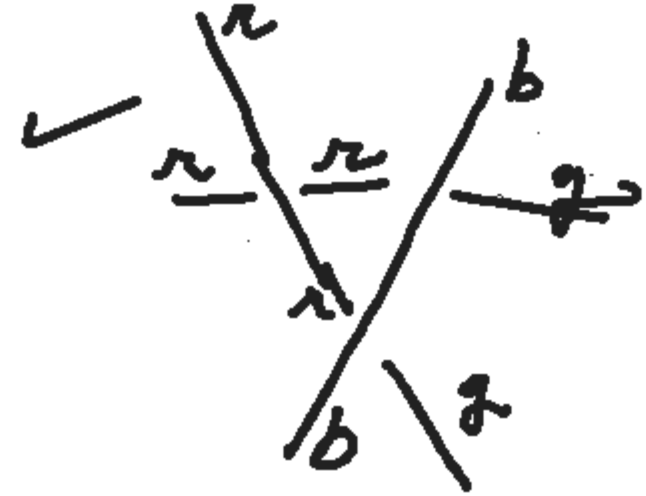


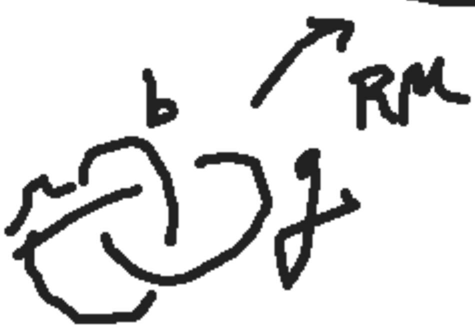
2



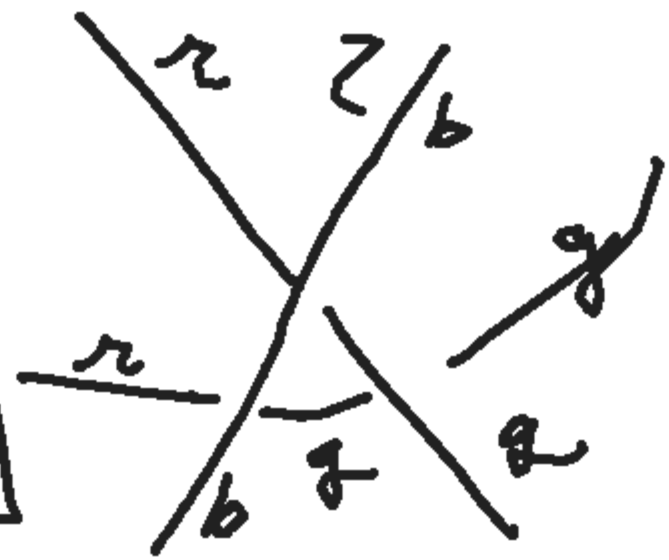
1. $xx = x$
2. $(xy)y = x$
3. $(xy)z = (xz)(yz)$

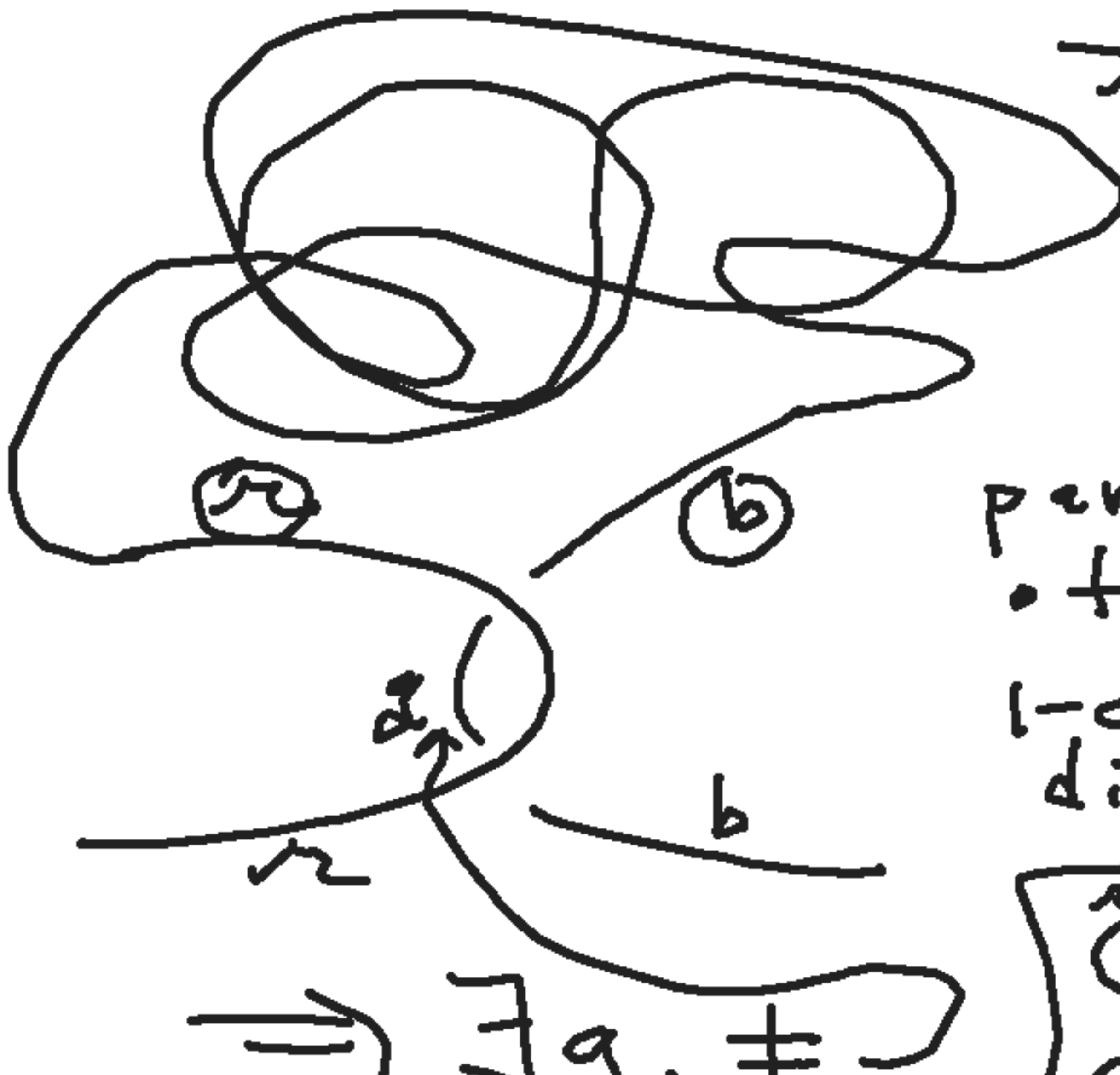






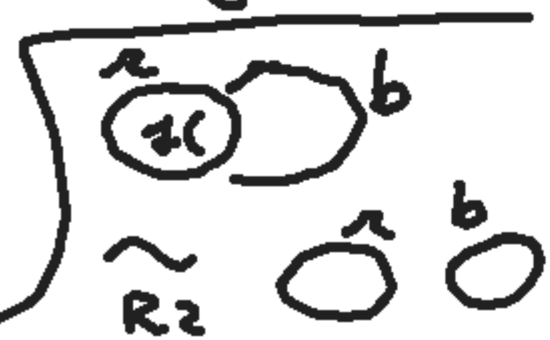
Ex: no colour knots on a knot



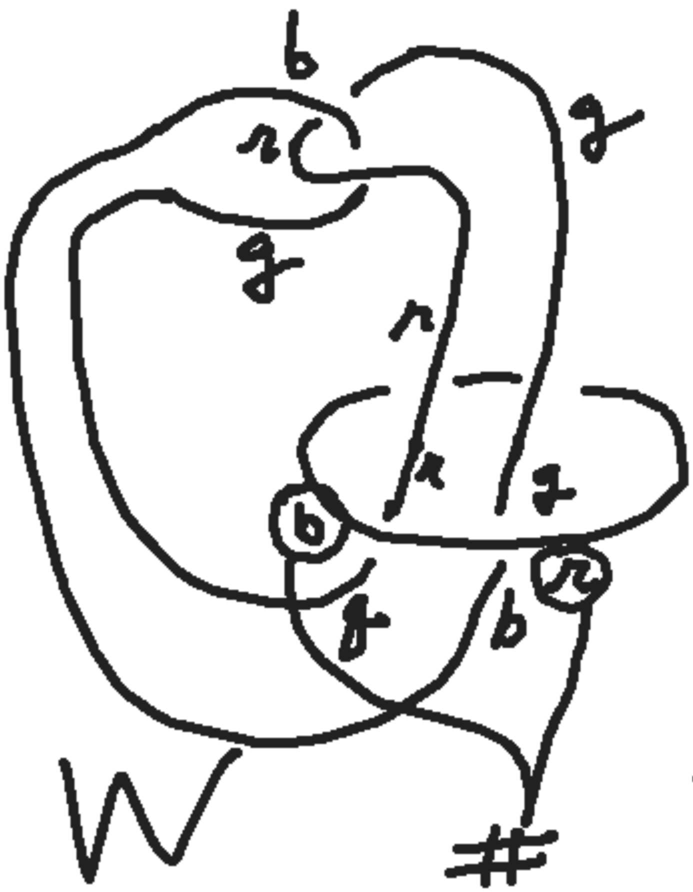


part
of a
1-component
diag.

$\Rightarrow \exists g \neq$



links can have
solutions.



W

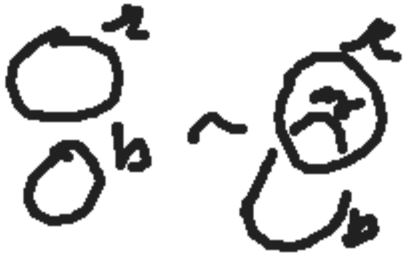
$\sim$
RM

Fact
W is
uncol
xcept
by 1
color.

$\Downarrow$

W
linked

Then
W can be
nontrivially
3 colored.



$\sim \ldots\ldots \sim$ L ≥ 2 colors