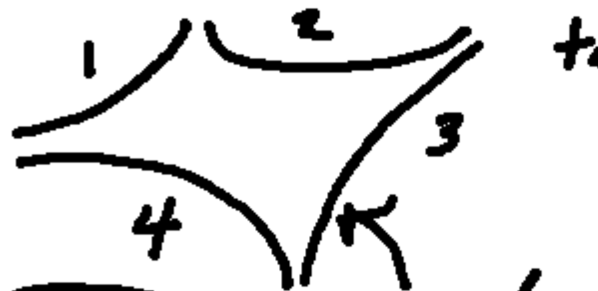
- One more lecture in the course next week.
- A problem set will be distributed in few days.
- We may continue in January some form.

1. Tubing Relation



$$\left.\begin{array}{l} \text{tubes}\ 1\ 2\ 3\ 4 \\ -\,1\ 2\ 3\ 4 \\ +\,1\ 2\ 3\ 4 \\ -\,1\ 2\ 3\ 4 \end{array}\right\} = \phi$$

$\boxed{\dfrac{\text{Cob}}{\text{Tube Reln}}}$ correct background for Kh^*_o.

$-$ 

$+ \cdots = \phi$

$$\partial(A,B,C,D) = (\overset{\frown}{AB}, C, D) \quad \leftarrow \text{a "product"}$$
$$- (A, \overset{\frown}{BC}, D)$$
$$+ (A, B, \overset{\frown}{CD})$$
$$- (\overset{\frown}{A, B, C, D})$$

$$\partial(A,B,C,D) = (AB, C, D)$$
$$- (A, BC, D)$$
$$+ (A, B, CD)$$
$$- (DA, B, C)$$

What is needed for $\partial \circ \partial = \phi$?

$$\partial\partial(A,B,C,D)$$

$$= \partial\big[(AB,C,D) - (A,BC,D) + (A,B,CD) - (DA,B,C)\big]$$

$$\begin{aligned}
= {} & ((AB)C,D) - (A(BC),D) + (AB,CD) \\
& - (AB,CD) + (A,(BC)D) - (A,B(CD)) \\
& + (D(AB),C) - (DA,BC) + ((CD)A,B) \\
& - ((DA)B,C) \\
& + (DA,BC) \\
& - (C(DA),B)
\end{aligned}$$

$= \emptyset$ empty when op. $A,B \to AB$ assoc.

$(A\ B)\ C \equiv A\ (BC)$

Tube Relation looks like setting $\partial\alpha = \emptyset$ in a complex made from cobordism.

But at this stage the boundary map is not quite alg. defined.

"$\text{Cobs}/\partial\,\text{Cobs}$"

What will come of further thought along this line?

The 4 Tube Relation is somehow analogous to the (famous) <u>4 Term Relation</u> for Vassiliev Invariants for Knots and Links.

An invar I of knots / links is said to be a <u>Vassiliev Invariant</u> if

I  $-\ I$  $=\ I$ 

singular crossing

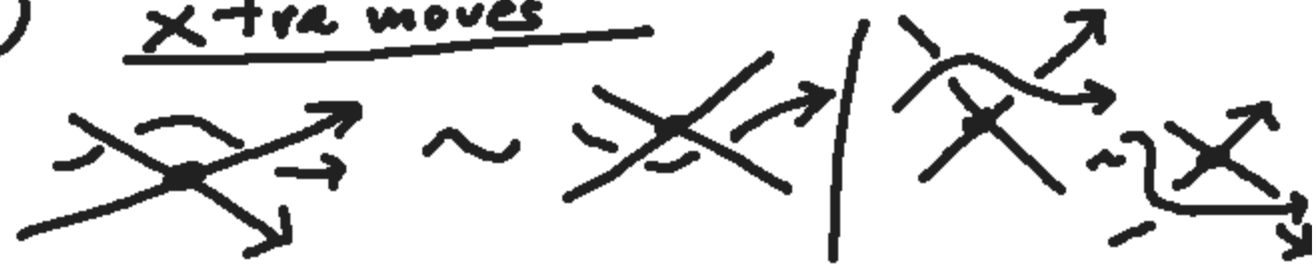
The invar is defined on knots with sing crossings .

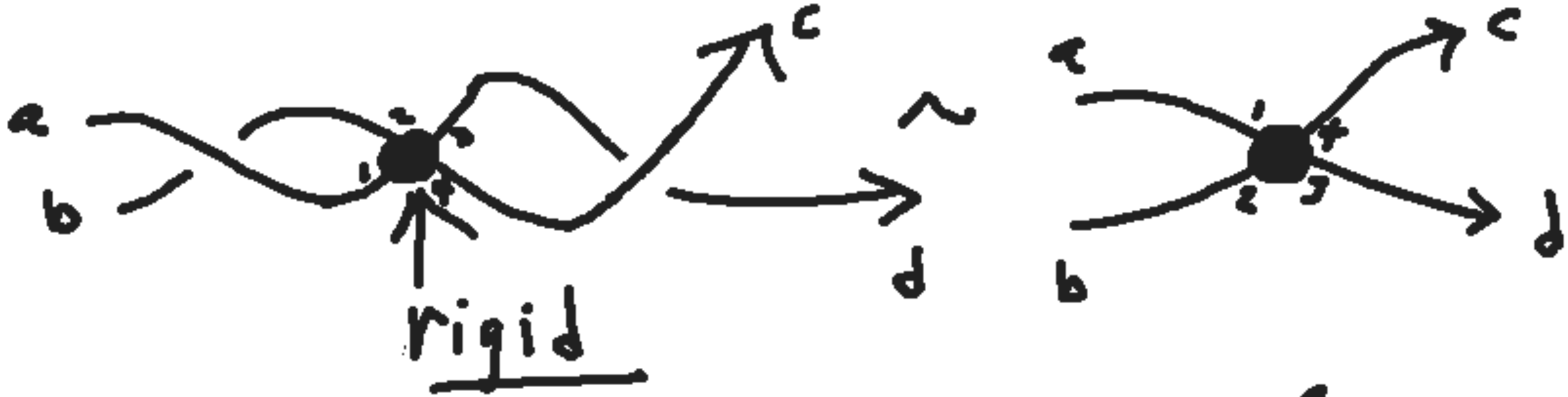
graphical

$\boxed{\text{Graphs} \subset \mathbb{R}^3}$

I 

<u>xtra moves</u>



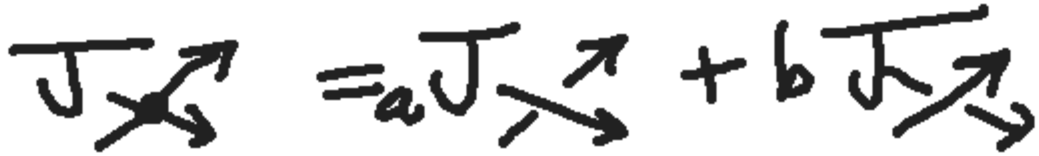


won't cancel

Rigid Vertex Moves

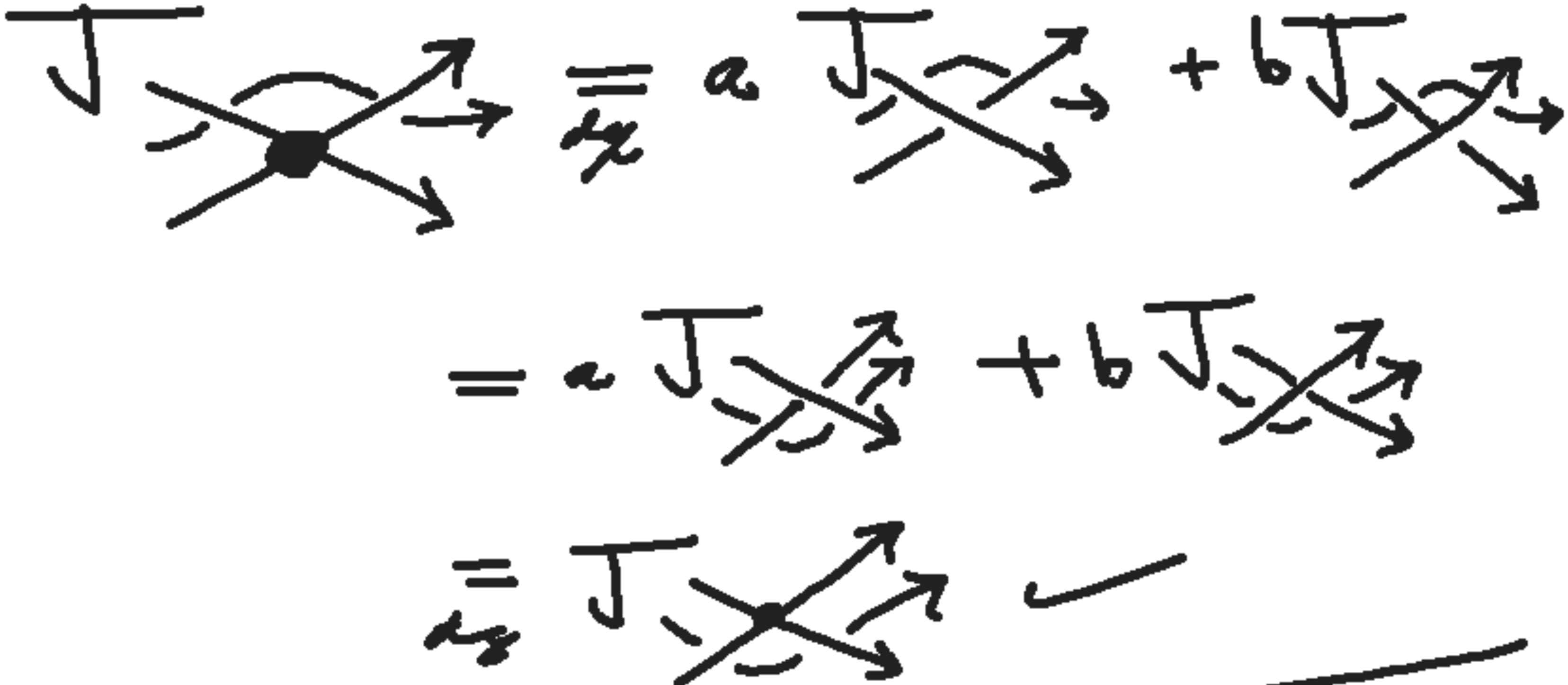
Fact. If J_K is an invar of knots & links then we extend J to RVGraphs via

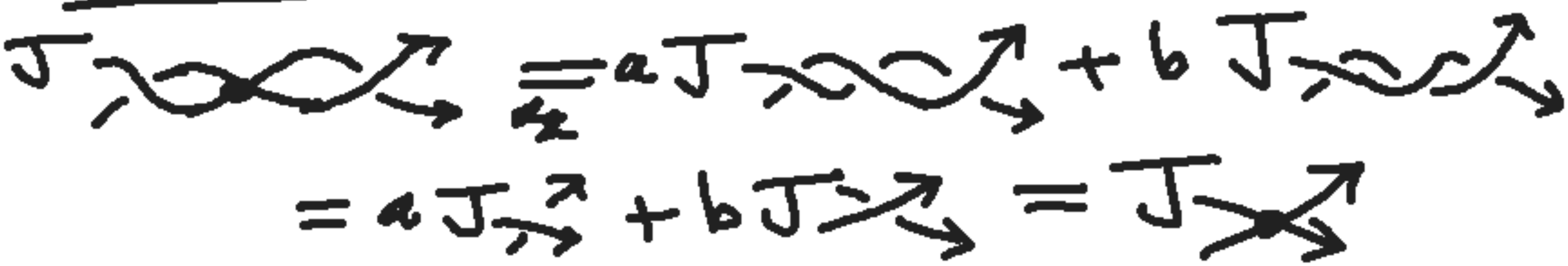
• J extended is an RVG invar.



eval J on the knots & links.

e.g. Let $J_K(q)$ = Jones poly.





$= a$ J 

$+ b$ J 

?

$\Rightarrow D$ is nontrivial RVG $\not\leftrightarrow D^*$

- I  $-\, I$  $= I$ 
- I is said to be of finite type N

If I  ...  $= \emptyset$. vanishes for G with $> N$ nodes.

$> N$

Lemma. If I is Vassiliev invar of type N then I_G is independent of $G \hookrightarrow \mathbb{R}^3$ for G with exactly N nodes.

$\boxed{N=2}$ I  $-$ $\cancel{I}$  $=$ I 

ϕ

Could assume

I  $= \phi$

I  $= \phi$

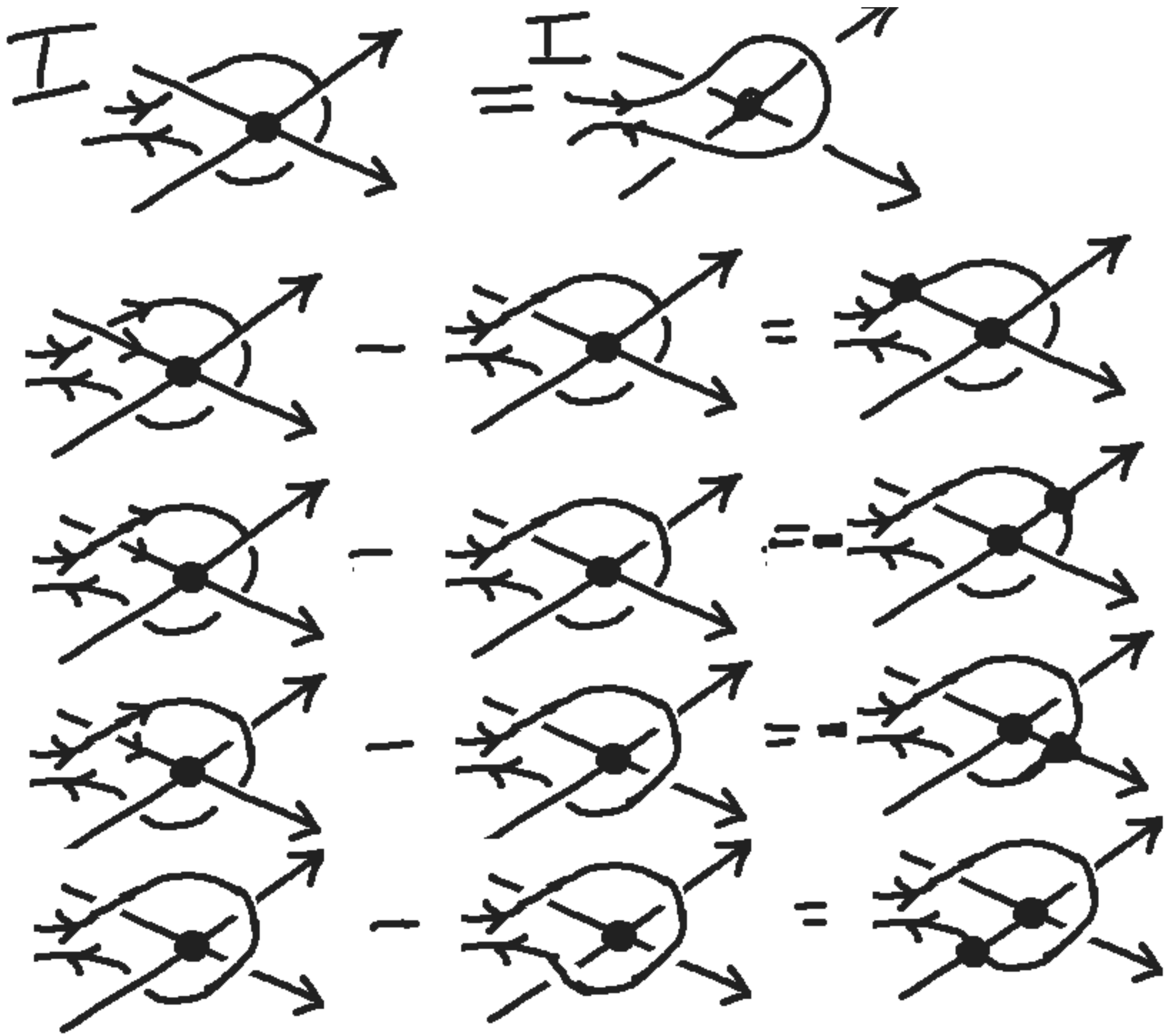
" I  $-$ I 

same I

I  $-$ I 

$= I$  $= \phi$

Type 2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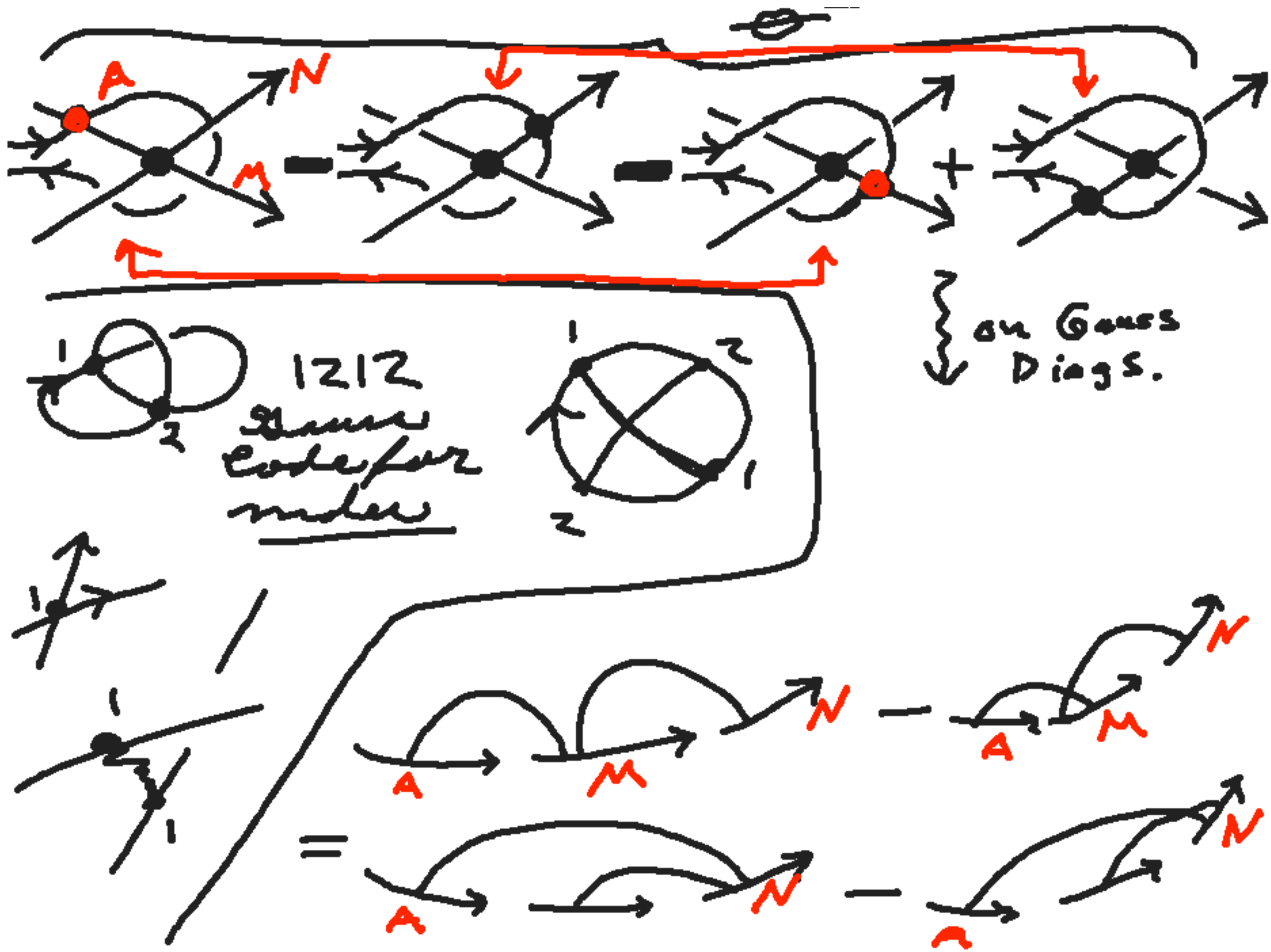
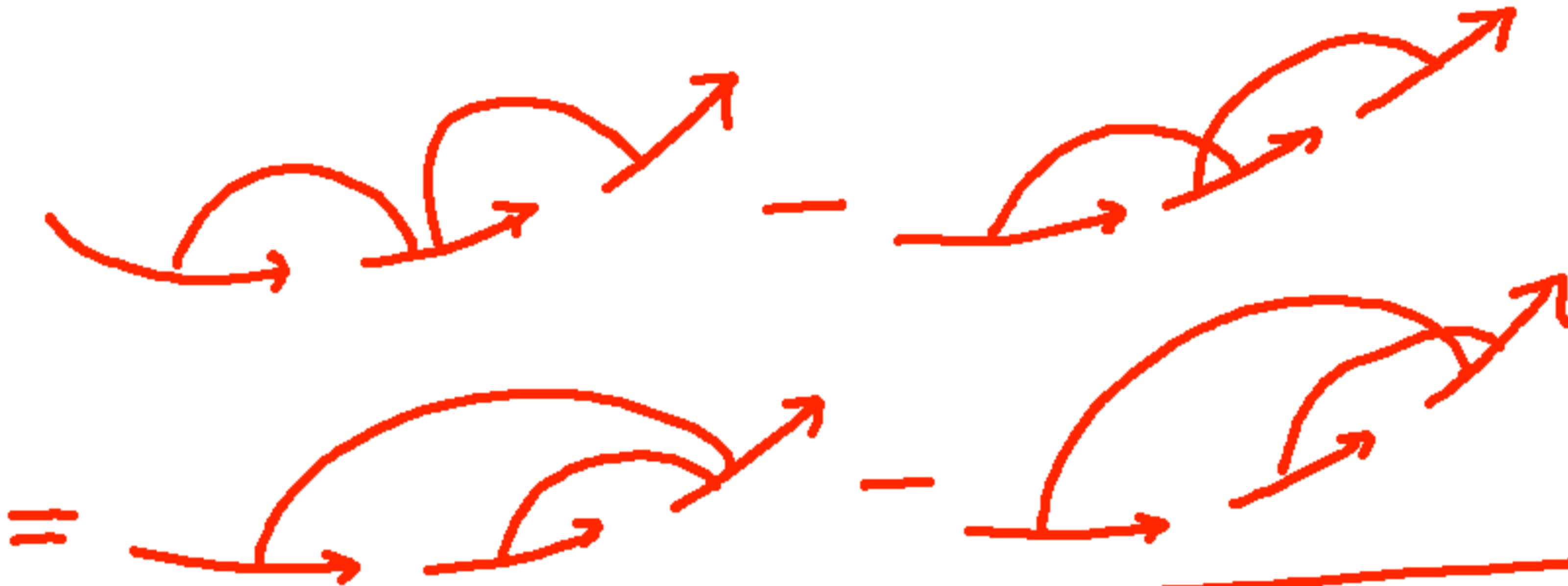


A
N
M
on Gauss Diags.
1212
Gauss Code for index
1
2
=



Diag Matrices

sum over internal indices

$$\sum_j T_{ij} S_{jk} = (TS)_{ik}$$

Lie Alg $\{T^a\}_{a=1,\dots,d}$ basis : $T^aT^b - T^bT^a = \sum_i f^{ab}_i T^i$

$[T^a, T^b] \in$ Alg gen by $\{T^c\}$

$$T^a T^b - T^b T^a = \sum_c f^{ab}_c T^c$$

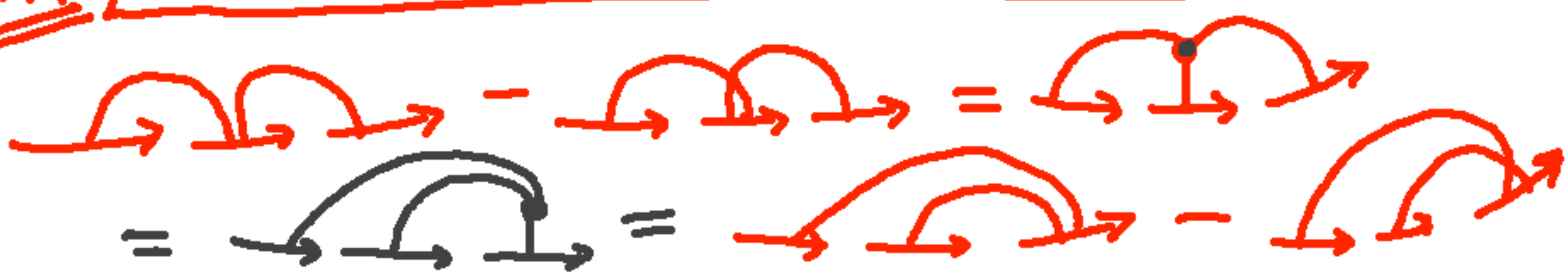
Let's assume Y^{ab}_c invar under perm

 $\equiv f^{ab}_c$

Lie Alg

$\Downarrow$

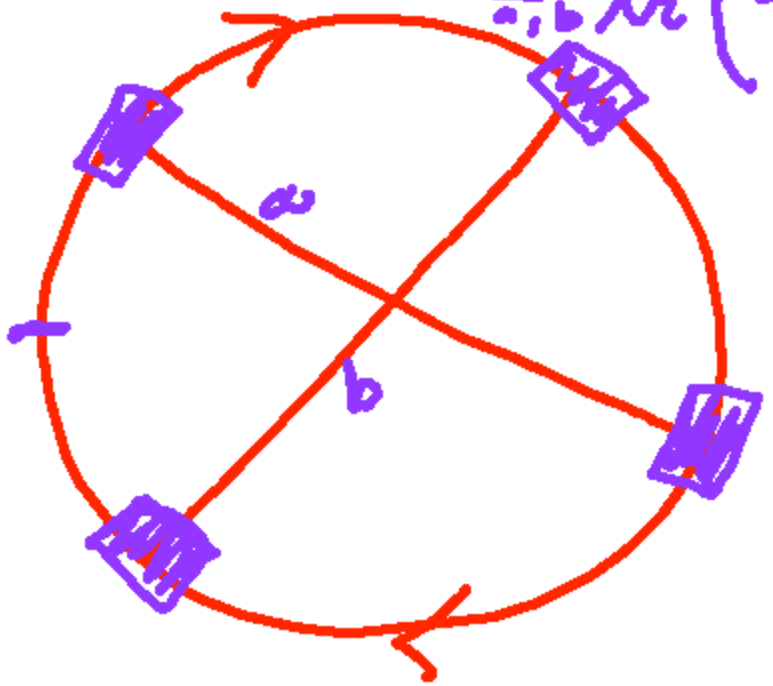
4TR

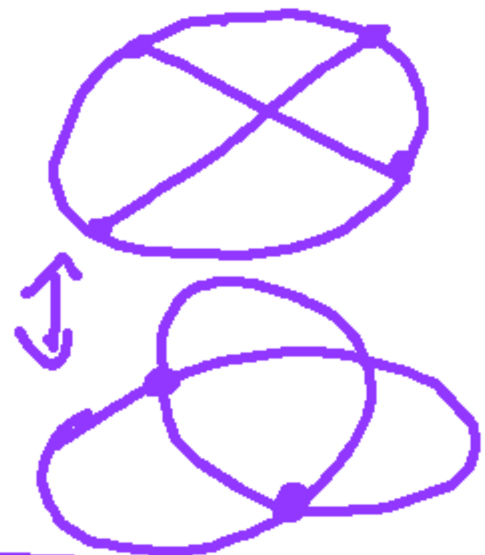


$$\sum_{a,b} \mathrm{tr}\left(T^a T^b T^a T^b\right)$$

$$\|$$

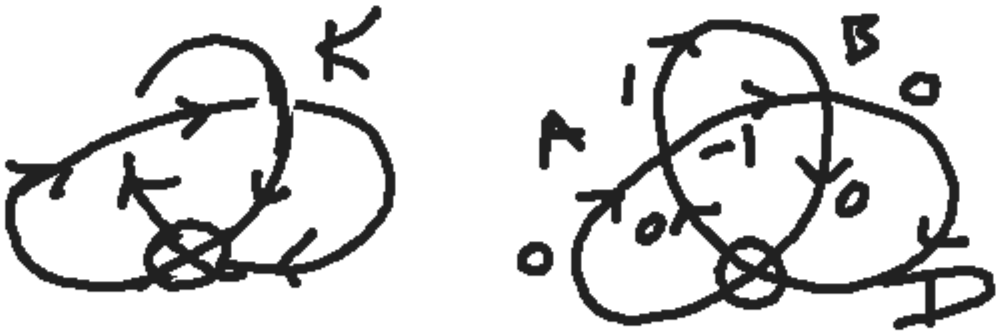
eval of





Digression on Affine Index

$Wt(D)$



	w_+	w_-
A	-1	1
B	1	-1

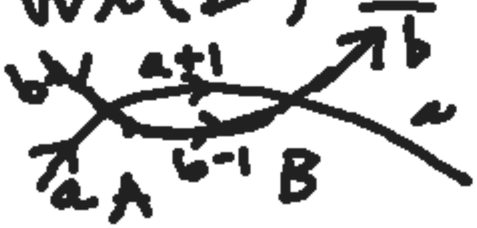
$$P_K = \sum_c \mathrm{sgn}(c)\left(t^{w_{\mathrm{sgn}(c)}} - 1\right)$$

$$P_K = t^{-1} + t - 2$$

4(D)

inv

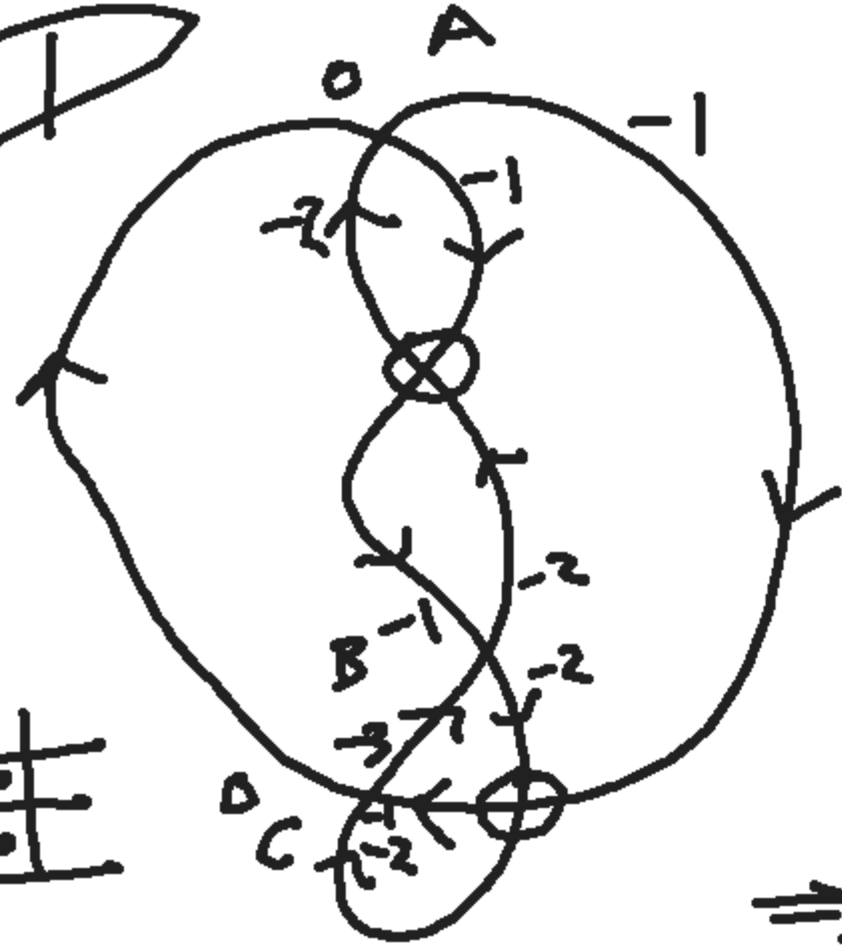
Wts Flat Virtuals

- $Wt(D) = Wt(D')$ if $D \underset{R3}{\sim} D'$
- 

	w_+	w_-
A	•	*
B	*	•

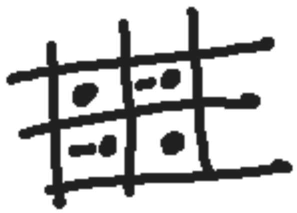
A	$b-a-1$	$a-b+1$
B	$a+1-b$	$b-1-a$

D



	w_+	w_-
A	1	-1
B	1	-1
C	-2	2

$\}\ W(D)$



<u>Fact</u> : D, D^r

$D^r = D$ with reversed orientation.

$\Rightarrow Wt(D^r) =$ Column Rersal of $Wt(D)$

Let $\hat{W}(D) =$ reversal of $W(D)$

<u>R3</u>. $W(D) \neq \hat{W}(D)$ preserved.

<u>R2</u>. $W(D) \neq \hat{W}(D)$ preserved

If $W(D) \neq \hat{W}(D)$ $\Rightarrow$ D not equiv to D^r.