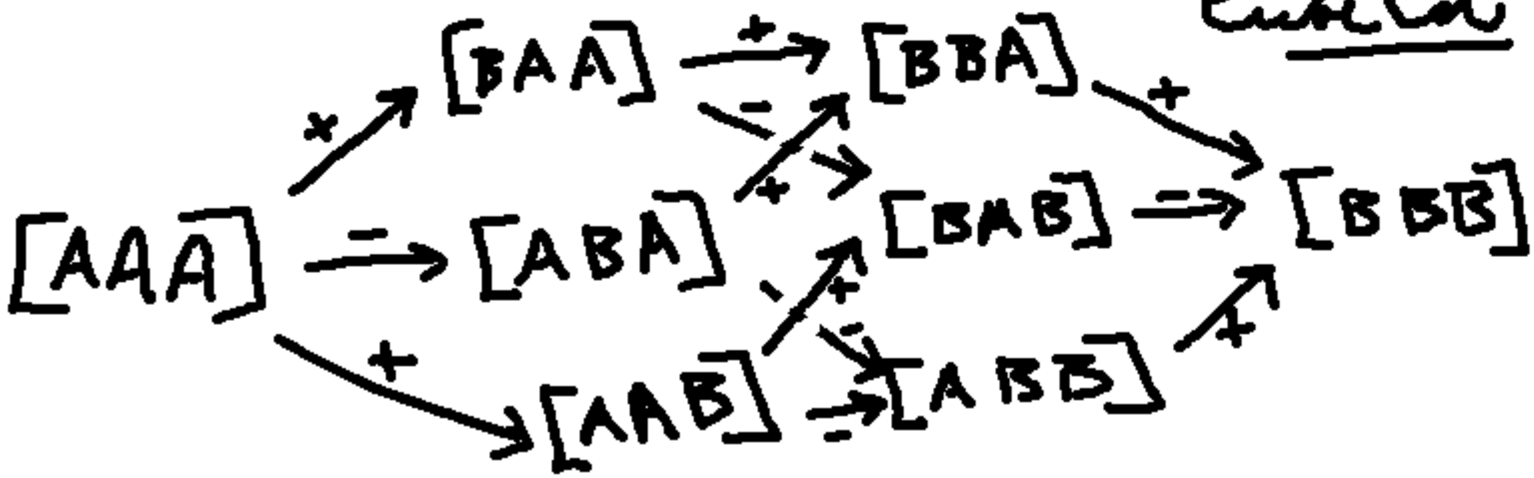
<u>Khovanov Homology</u>

The notes that follow are supplemental to the slide show.

Cube Cat



$$[\cdots A \cdots] \xrightarrow{\partial} [\cdots B \cdots]$$

$(-1)^{\#\text{ of } A\text{'s preceding the given } A}$

$$\Rightarrow \partial \circ \partial = 0 \text{ over } \mathbb{Z}.$$

$$(A+B)^2 = A^2 + 2AB + B^2$$

$$\begin{array}{c|c|c} & A & B \\ \hline A & A^2 & AB \\ \hline B & BA & B^2 \end{array}$$

$$(A \to B)^2 = (A \to B)(A \to B)$$

(<u>distributive law</u>)

$$= (A \to B)A \longrightarrow (A \to B)B$$

$$= (AA \to BA) \longrightarrow (AB \to BB)$$

a higher category
(∃ morphisms between morphisms)

$$= \begin{tikzcd} AA \arrow[r] & BA \\ AB \arrow[r] \arrow[u, phantom, "\Downarrow"] & BB \end{tikzcd}$$

$$\xrightarrow{\mathcal{F}} \begin{tikzcd} AA \arrow[r] \arrow[d] & BA \arrow[d] \\ AB \arrow[r] & BB \end{tikzcd}$$

<u>Flatten</u>

$$\begin{tikzcd} \sigma \arrow[r] & \sigma' \\ \rho \arrow[r] \arrow[u, phantom, "\Downarrow"] & \rho' \end{tikzcd} \rightsquigarrow \begin{tikzcd} \sigma \arrow[r] \arrow[d] & \sigma' \arrow[d] \\ \rho \arrow[r] & \rho' \end{tikzcd}$$

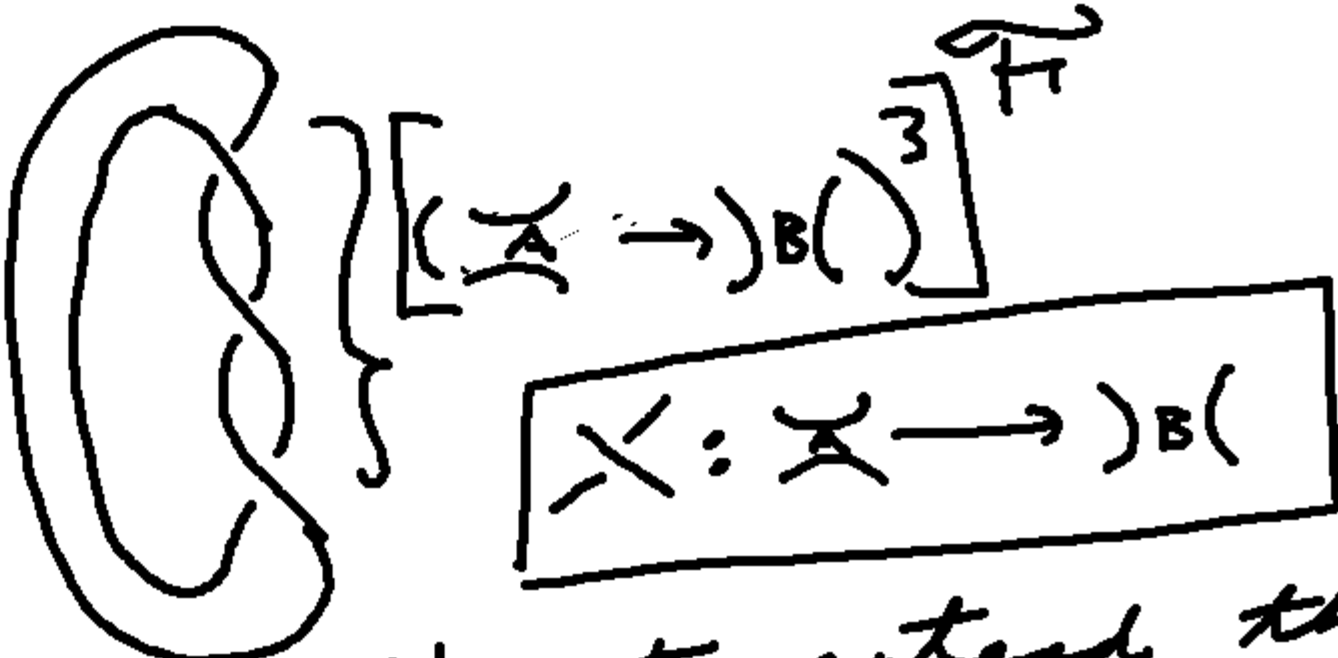
commute

$$(A \to B)^3 = (A \to B)\left[\begin{array}{ccc} AA & \longrightarrow & AB \\ & \downarrow & \\ BA & \longrightarrow & BB \end{array}\right]$$

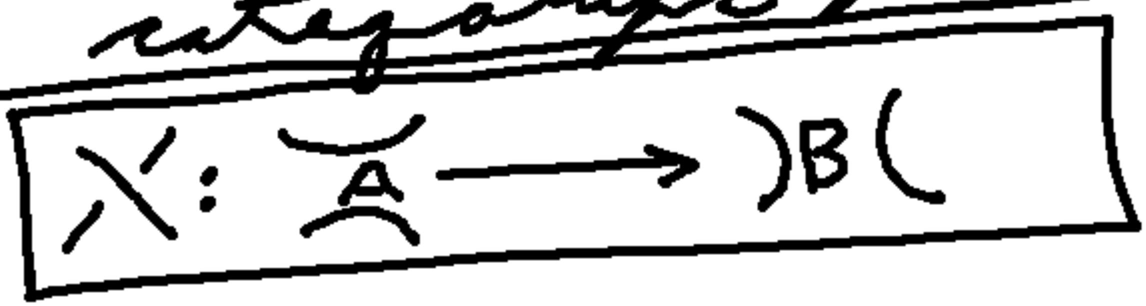
$$= \begin{array}{ccc} AAA & \longrightarrow & AAB \\ & \downarrow & \\ ABA & \longrightarrow & ABB \end{array} \longrightarrow \begin{array}{ccc} BAA & \longrightarrow & BAB \\ & \downarrow & \\ BBA & \longrightarrow & BBB \end{array}$$

$$\Big\downarrow F$$

<u>Cube Cat</u>



Hope to extend this idea to other categorifications of link invariants.



$f,g : \mathcal{C} \longrightarrow \mathcal{C}'$ chain complexes

$\mathcal{C}^n \xrightarrow{\partial} \mathcal{C}^{n+1}$

$\mathcal{C}'^n \xrightarrow[\partial]{} \mathcal{C}'^{n+1}$

$f \sim g$ chain homotopic

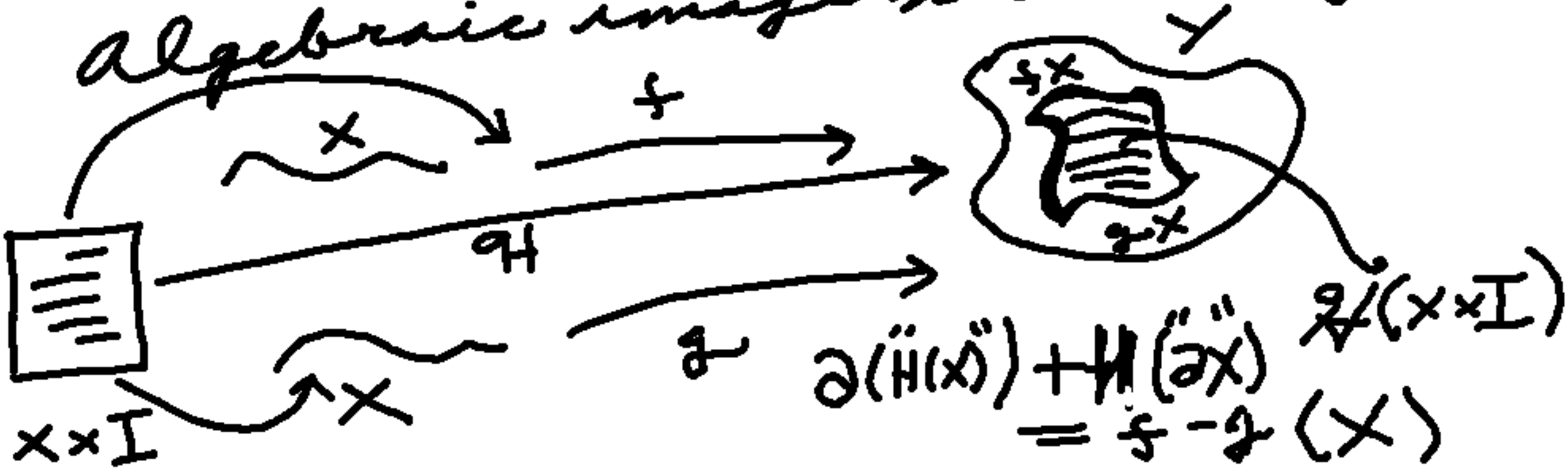
if $\exists\ H : \mathcal{C}^n \longrightarrow \mathcal{C}'^{n-1}$

$\partial \circ H + H \circ \partial = f - g$

$\mathcal{C}^n \xrightarrow[H]{} \mathcal{C}'^{n-1}$

$\Longrightarrow$ f + g induce same map on homology.

algebraic image of a homotopy.



$\partial(''H(X)'') \pm H(''\partial X'')$
$= f - g\,(X)$

We say $\mathcal{C}$ + $\mathcal{C}'$ are
<u>chain homotopy equivalent</u>

if $\exists$ $f : \mathcal{C} \to \mathcal{C}'$

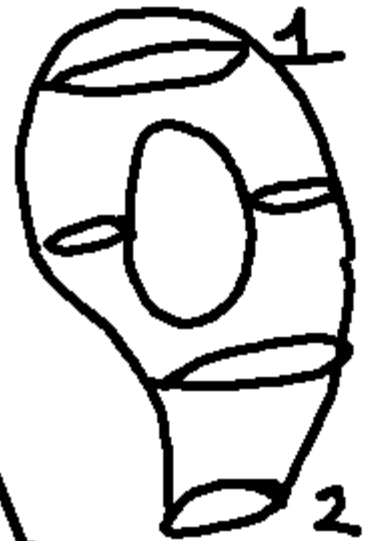
$g : \mathcal{C}' \to \mathcal{C}$

s.t $g \circ f \sim \underline{1}_{\mathcal{C}}$

$f \circ g \sim \underline{1}_{\mathcal{C}'}$

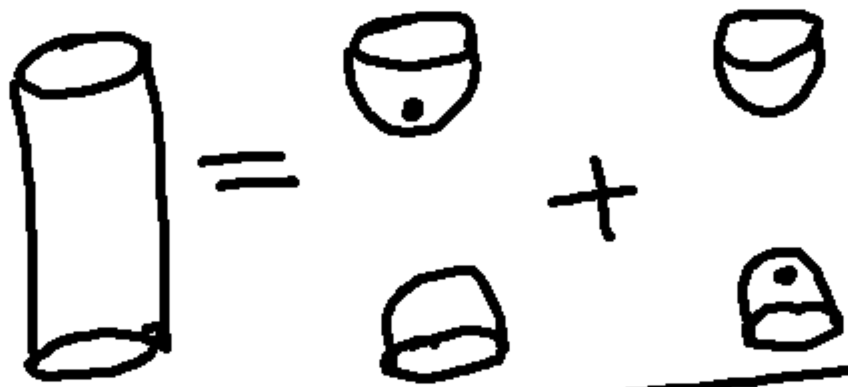
$\Rightarrow H(\mathcal{C}) \cong H(\mathcal{C}')$.

$\frac{1}{2}\Bigg($



$\Bigg)$

$=$



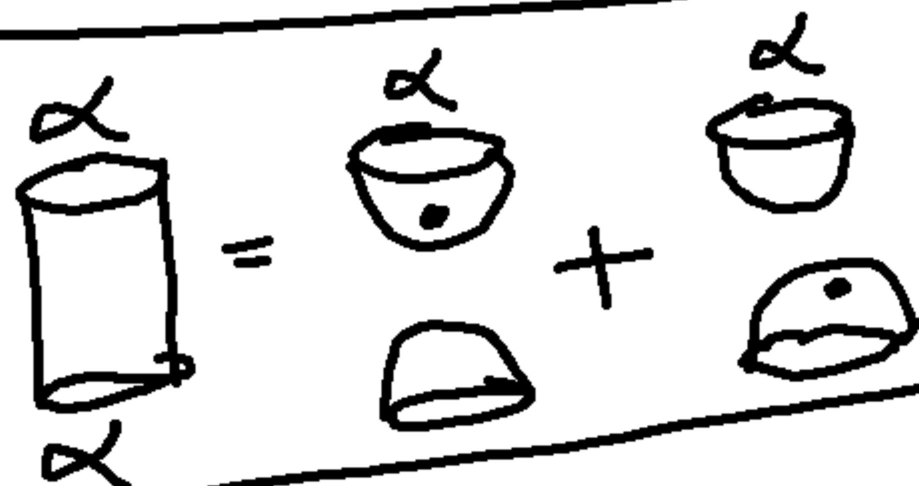
$\boxed{\bullet \equiv x}$

$1 = \varepsilon(x)1 + \varepsilon(1)x$

$1 = 1\cdot 1 + 0\cdot x$ ✓

$x = \varepsilon(x^2)1 + \varepsilon(x)x$

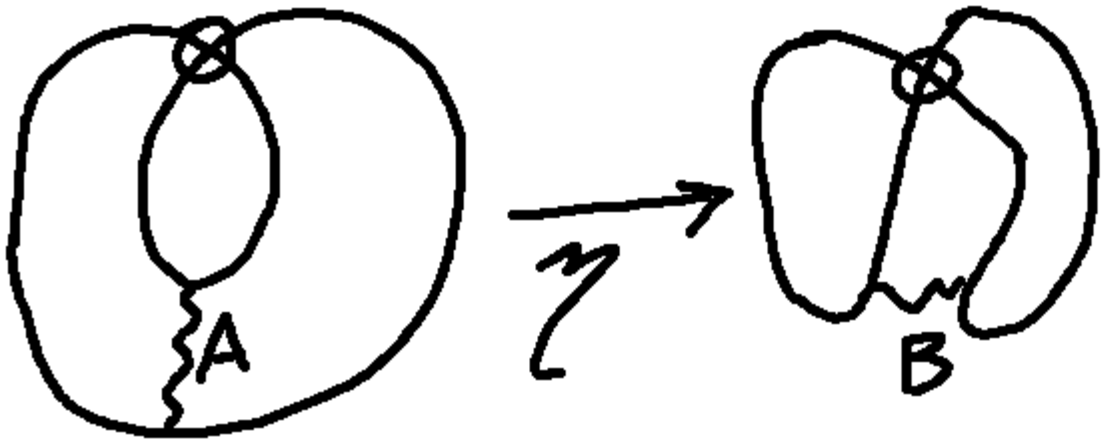
$x = 0 + 1\cdot x$



$$\boxed{\alpha = \varepsilon(\alpha x)\mathbb{1} + \varepsilon(\alpha)x}$$

correct in our algebra

Tube Relation is true.



? what to do?

1) $\eta = 0$ & make some new rules.

2) change something else (Rushworth)

Doubled Khovanov Homology