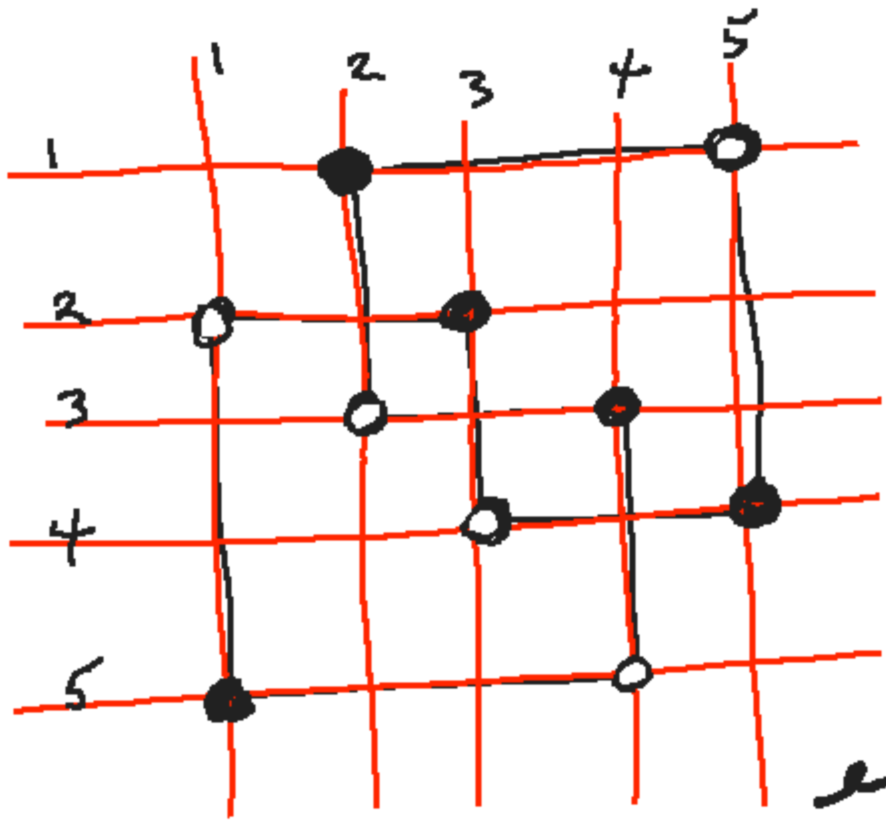
Ozsvath + Szabo

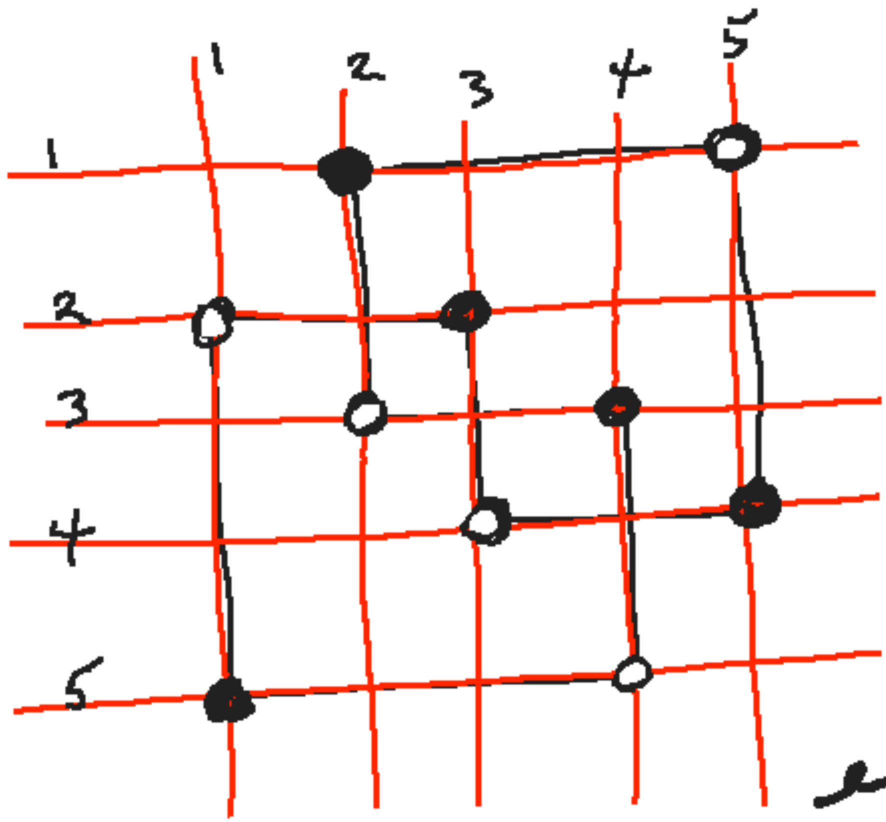
Knot Floer Homology

based comb on Rect Diags



encodes knots

$$\left\{ \begin{array}{l} B\text{perm}: \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \end{pmatrix} \\ W\text{perm}: \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 1 & 2 & 3 & 4 \end{pmatrix} \end{array} \right\}$$



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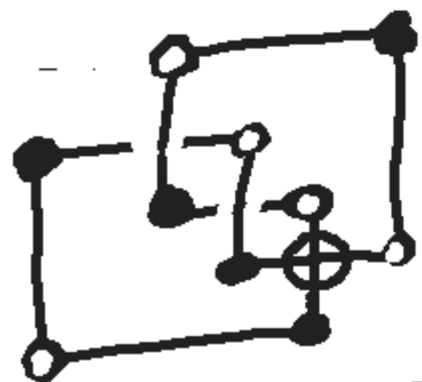
Program "GridLink" Python Marc Culler at UIC.

copy for Mac in our dropbox

1) Any tame knot diagram can be put in Rect form. (exercise!)

+ 2perms

2) How would you do a theory of virtual Rect diags?



diagrams work. but not det by B/W

+ moves

Research Question

D

$(v=2)$

$4v = 2e$

$2v = e$

$(e=4)$

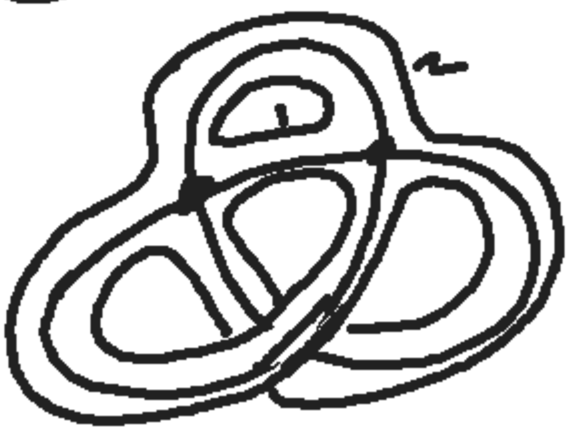
$g=1$

$v=1$
$e=2$
$f=1$

Note Interior of the face is a disk.

$v-e+f = 1-2+1 = 0$

$2-2g = 2-2 = 0$



add disks to boundary of ribbon graph

$\lambda = \#$ bndry components $(=2)$

$\Sigma(D) =$ surface of D

Euler Formula

For a surface

$v - e + f = 2 - 2g$

nodes, edges, disks

g = genus of surface

$v - 2v + \lambda = 2 - 2g$

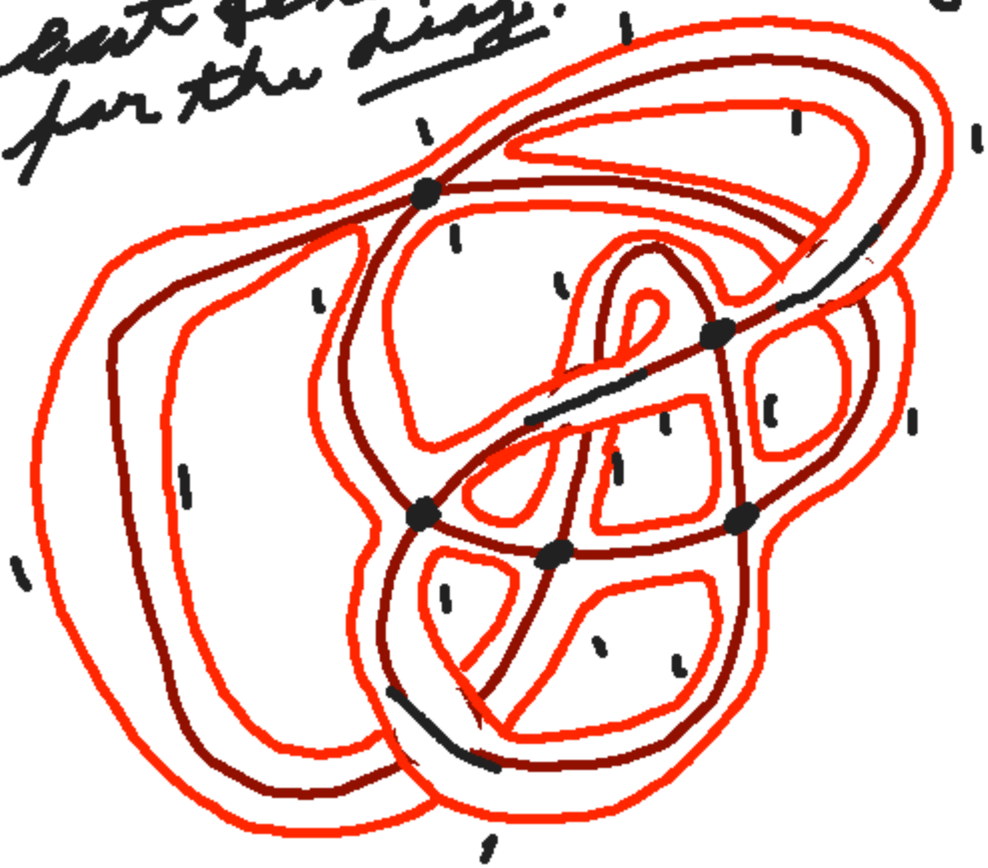
$-v + \lambda = 2 - 2g$

$2g - 2 = v - \lambda$

$$\boxed{g = 1 + \left(\frac{v-\lambda}{2}\right)}$$

$$\boxed{g = 1 + \left(\frac{v - \lambda}{2}\right)}$$

least genus for the diag.



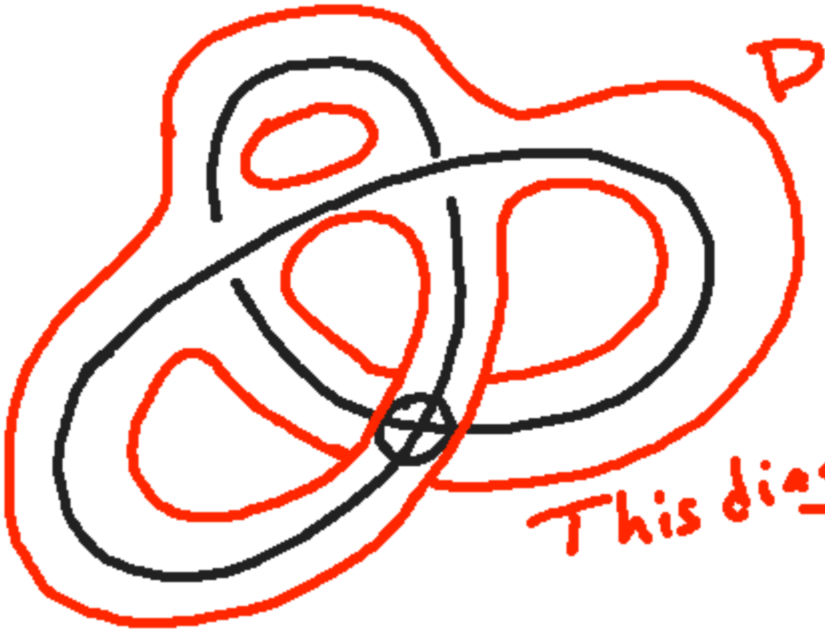
$v = 5$

$\lambda = 1$

$g = 1 + \frac{5-1}{2}$

$= 1 + 2$

$\underline{\underline{g = 3}}$



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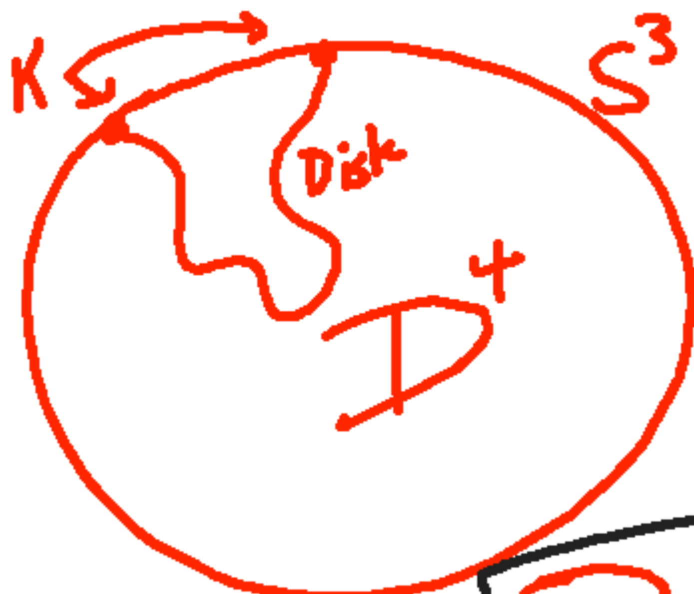
This diag ~ simpler diag D' of lower genus.

A classical knot is *slice* if it bounds a disk in 4-space.



4 ball whose bndry is S^3.

$T \# T^*$ bounds a disk in D^4.

concordance

not a concordance

$$S^3 \times I \subset D^4$$

$$S \subset S^3 \times I$$

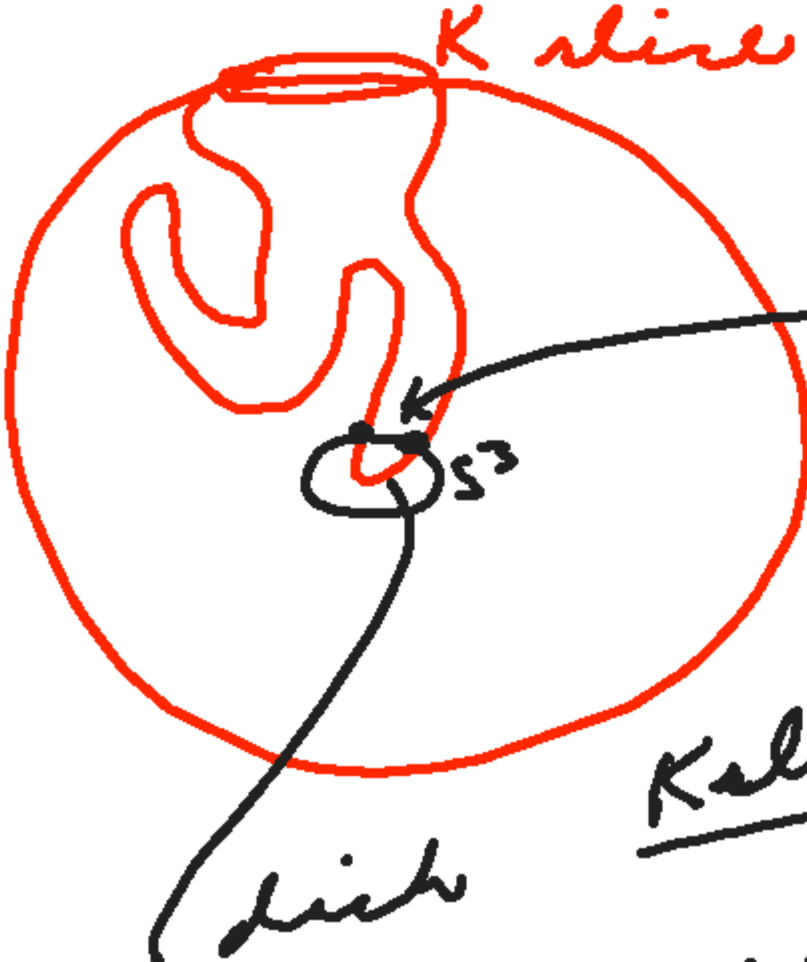
$$\partial S = K \cup K'$$

$K \subset S^3 \times 0$, $K' \subset S^3 \times 1$

S orientable

If genus(S) = 0 we say that S is a <u>concordance</u> between K and K'.

<u>K and K' are concordant.</u>



K slice $\Longleftrightarrow$ K concordant to $\bigcirc$ unknot.

Fact $K \# K^* \underset{c}{\sim} \bigcirc$

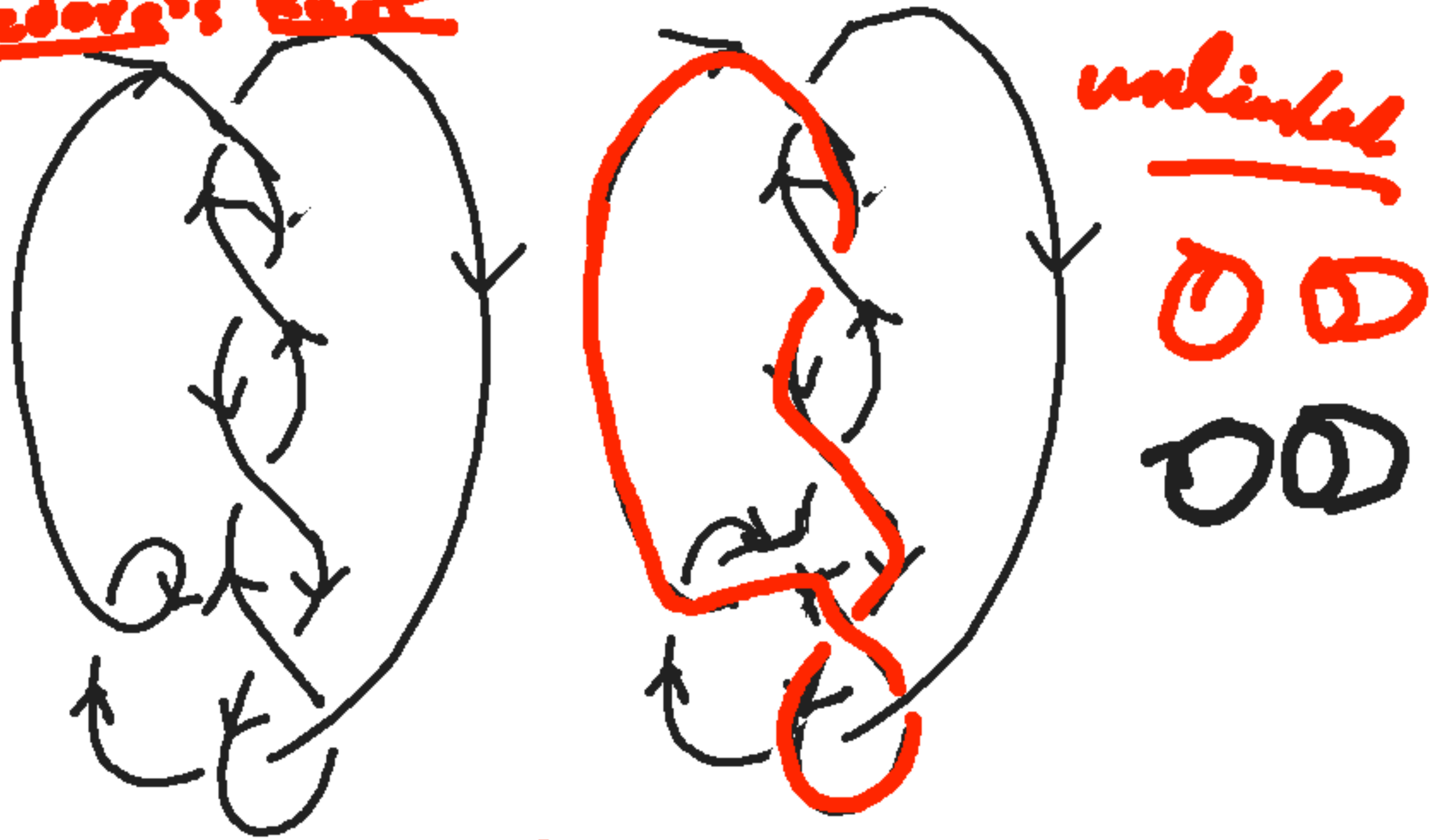
any classical K

If $K \underset{c}{\sim} K'$, $L \underset{c}{\sim} L'$ $\Rightarrow$ $K \# L \underset{c}{\sim} K' \# L'$

Concordance Classes of Knots Form a Group.

stevedore's knot

K



K slice $+$ $K \cong k\#l$