Course in Knot Theory

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Please send an email telling your interests.

assumption — small background
- abstract alg
- basic pt. set topology

Knot Projection + Reidemeister Moves

(representative)

A Knot is an embedding of a circle $S^1 \longrightarrow \mathbb{R}^3$ or S^3

$\mathbb{R}^3 = 3$ space $= \{(x, y, z) \mid x, y, z \in \mathbb{R}\}$

$\mathbb{R} =$ real numbers.

$S^3 =$ three sphere $= \{(x, y, z, w) \mid x^2 + y^2 + z^2 + w^2 = 1\}$

$\mathbb{R}^3 \subset S^3 \subset \mathbb{R}^4$

$S^3 \xrightarrow{\text{st}} \mathbb{R}^3$ Stereographic Proj

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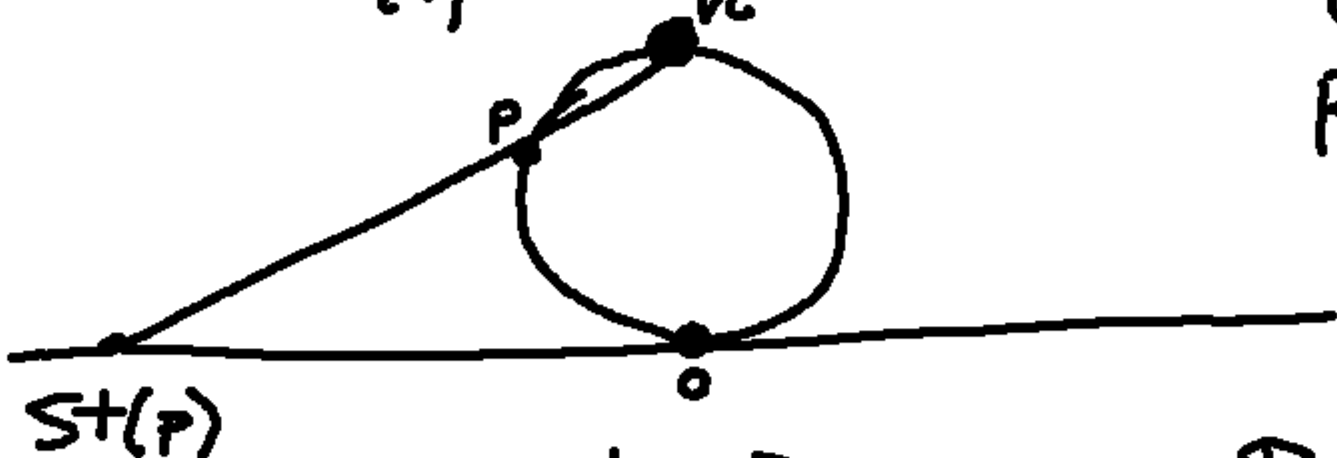
$S^3 \cong$ 1-point compactification of $\mathbb{R}^3$.

homeomorphic

$\mathbb{R}^1$, $S^1 - \{n\} \xrightarrow{St} \mathbb{R}$

$S^1 \subset \mathbb{R}^2 \supset \mathbb{R}$



$St : S^1 - \{n\} \longrightarrow \mathbb{R}$

<u>homeom</u>

$St : S^2 - \{n\} \to \mathbb{R}^2$

$\alpha: S^1 \to \mathbb{R}^3$
$\beta: S^1 \to \mathbb{R}^3$

Two knots are ambient isotopic if there is a continuous family γ_s $(s \in [0,1])$ of knots $\gamma_s : S^1 \to \mathbb{R}^3$ s.t.

$$\gamma_0 = \alpha$$
$$\gamma_1 = \beta$$

$$S^1 \subset \mathbb{R}^3$$

Knot Diagrams

T trefoil

E figure eight

K

For us the embedding are differentiable

or simply finite piecewise linear.

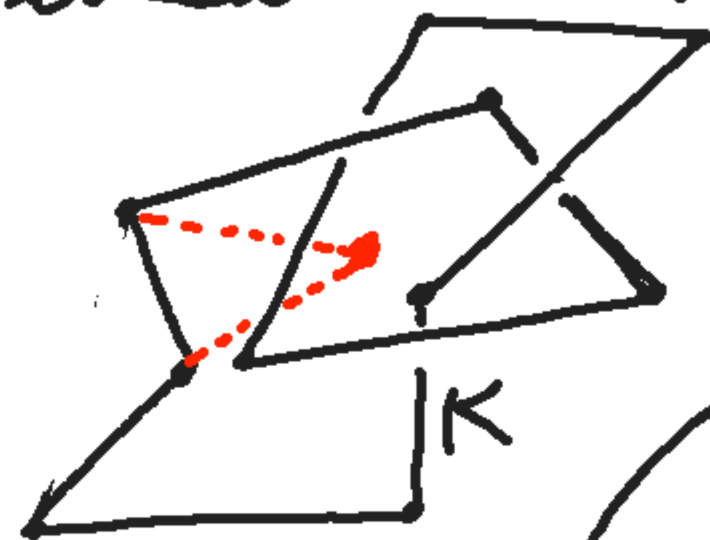
Not wanted

projection plane

Reidemeister/Alexander/Briggs

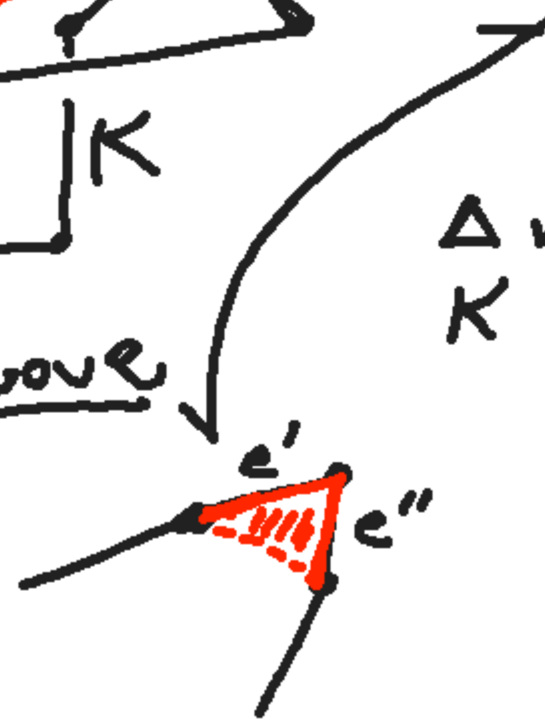
used PL (piecewise linear) ambient isotopy.

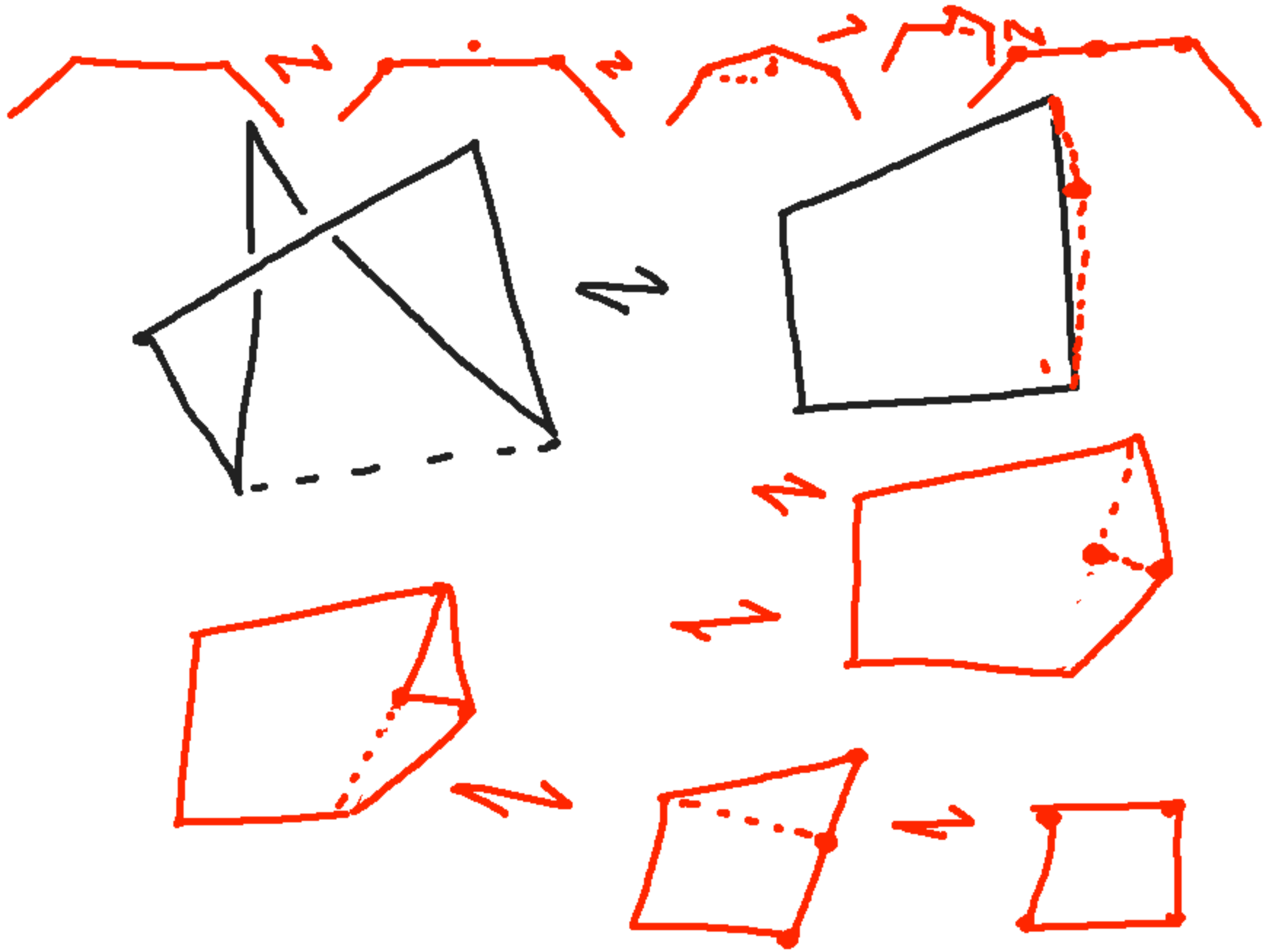
$K \underset{PL}{\sim} K'$
seq of
Δ-moves



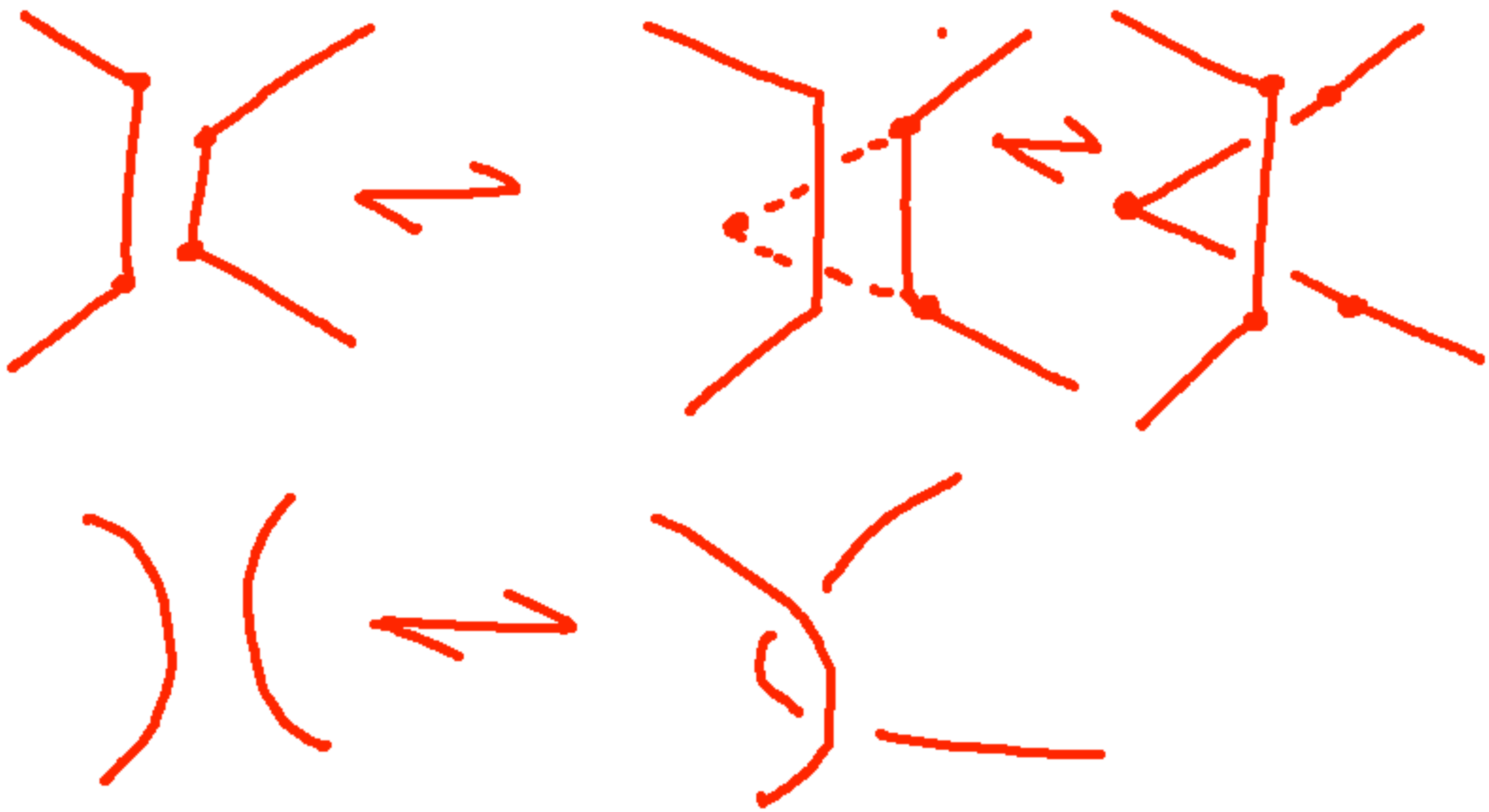
Δ must not intersect K except at e.

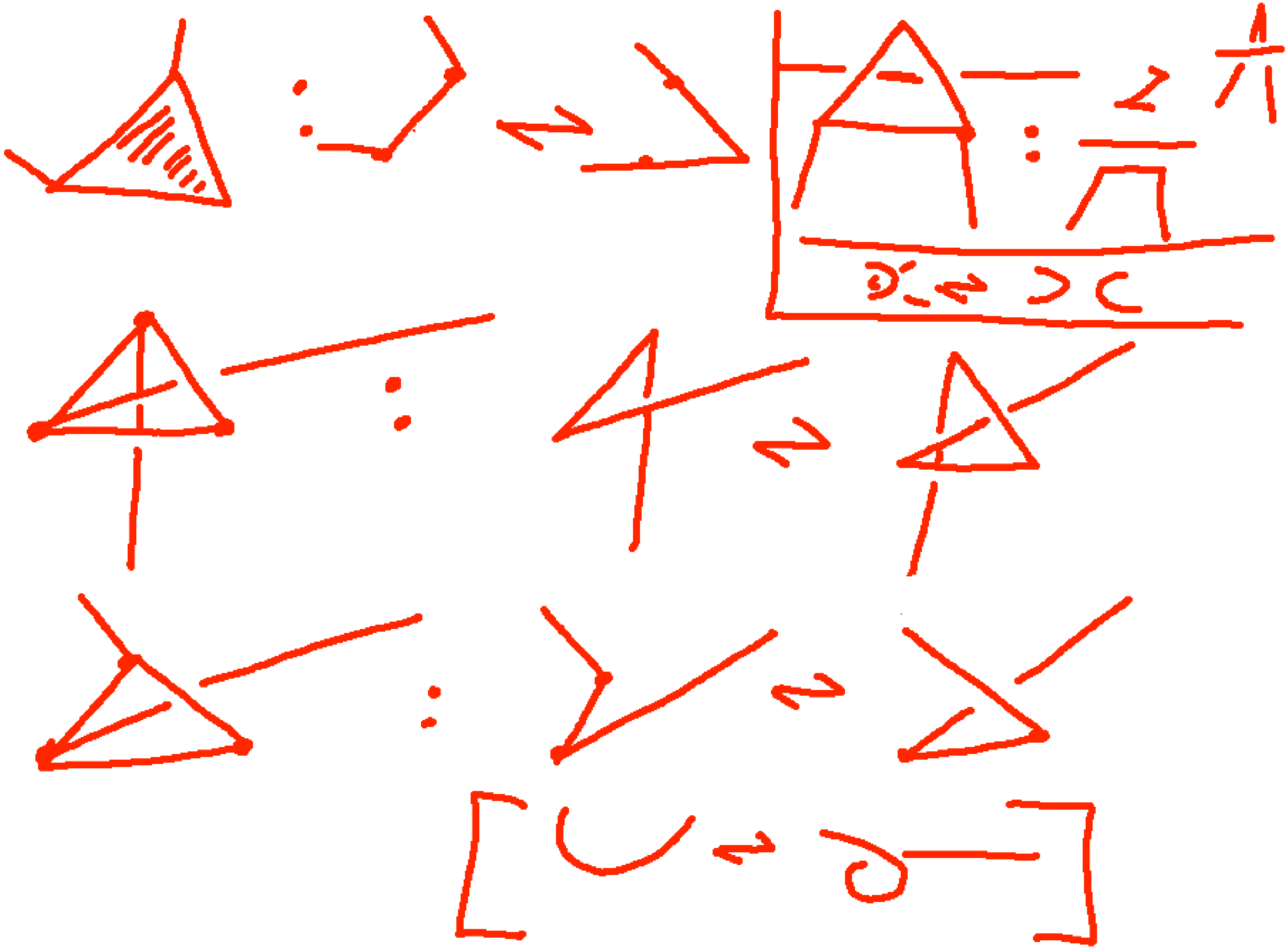
Triangle Move

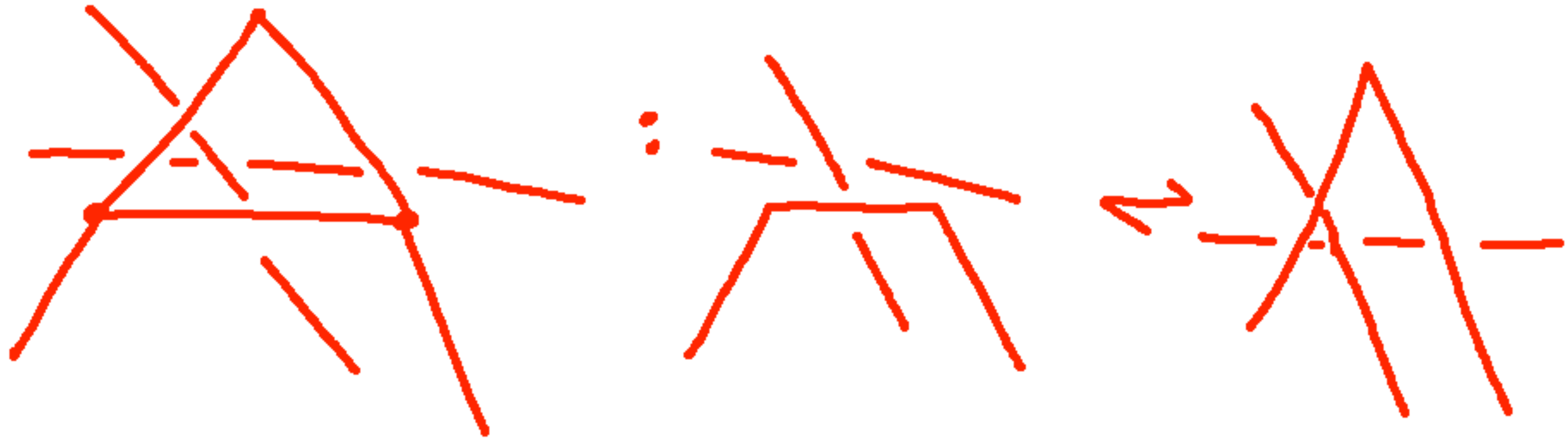




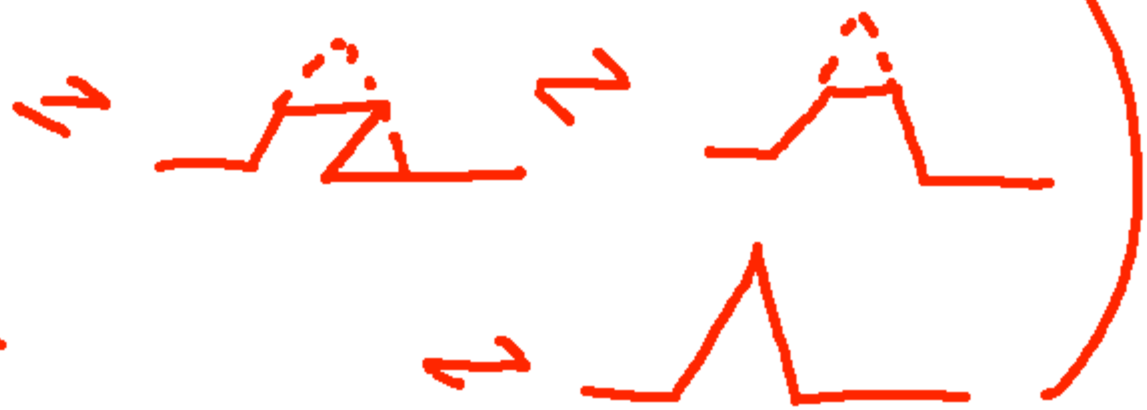
What do the projections of the Δ-moves amount to?







- We can subdivide any Δ as much as we like.



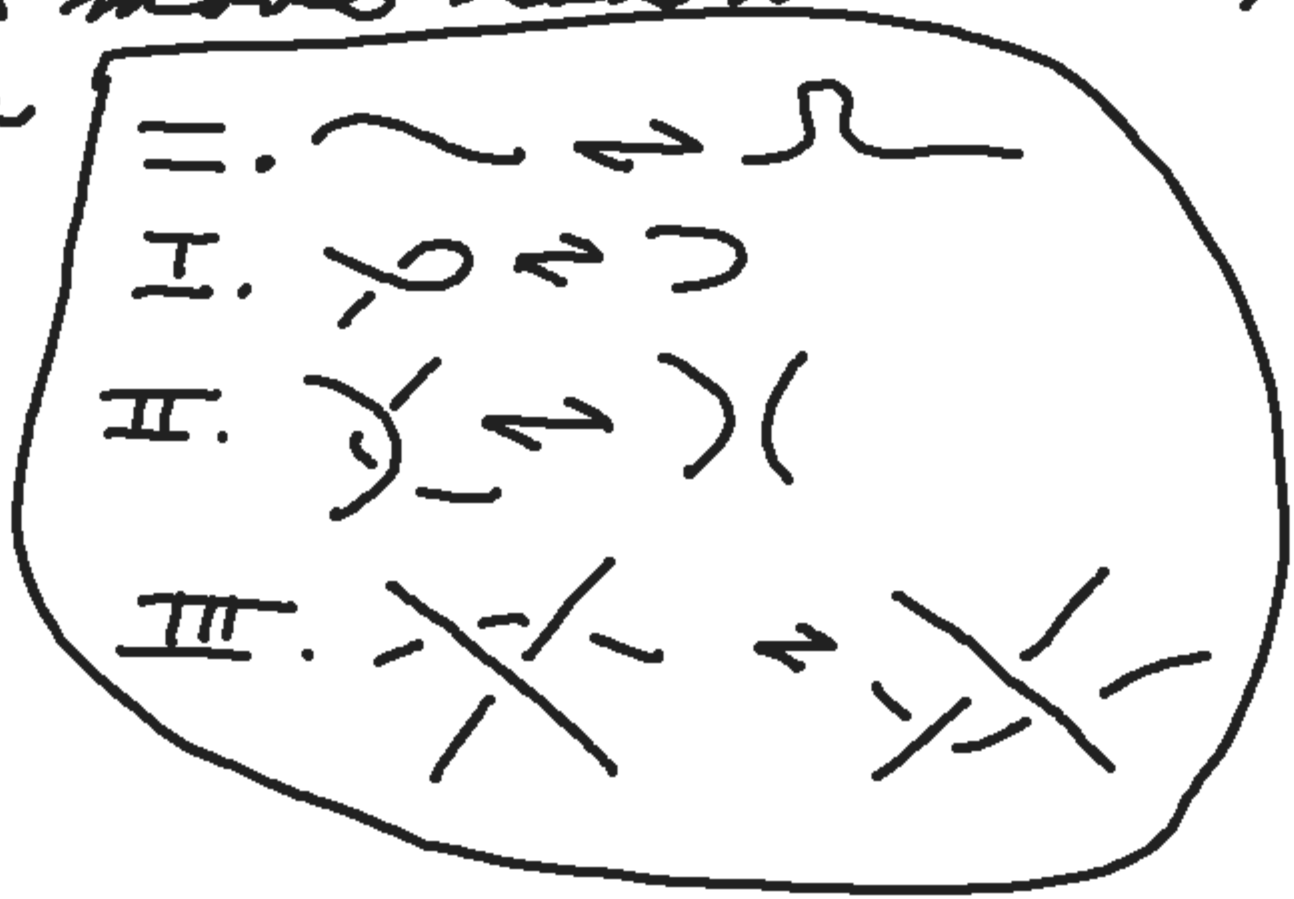
a very small Δ may contain one extra line

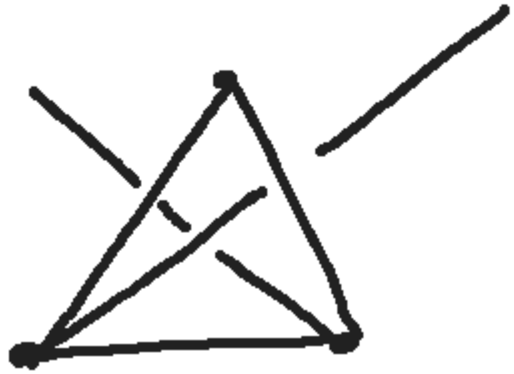
or a crossing

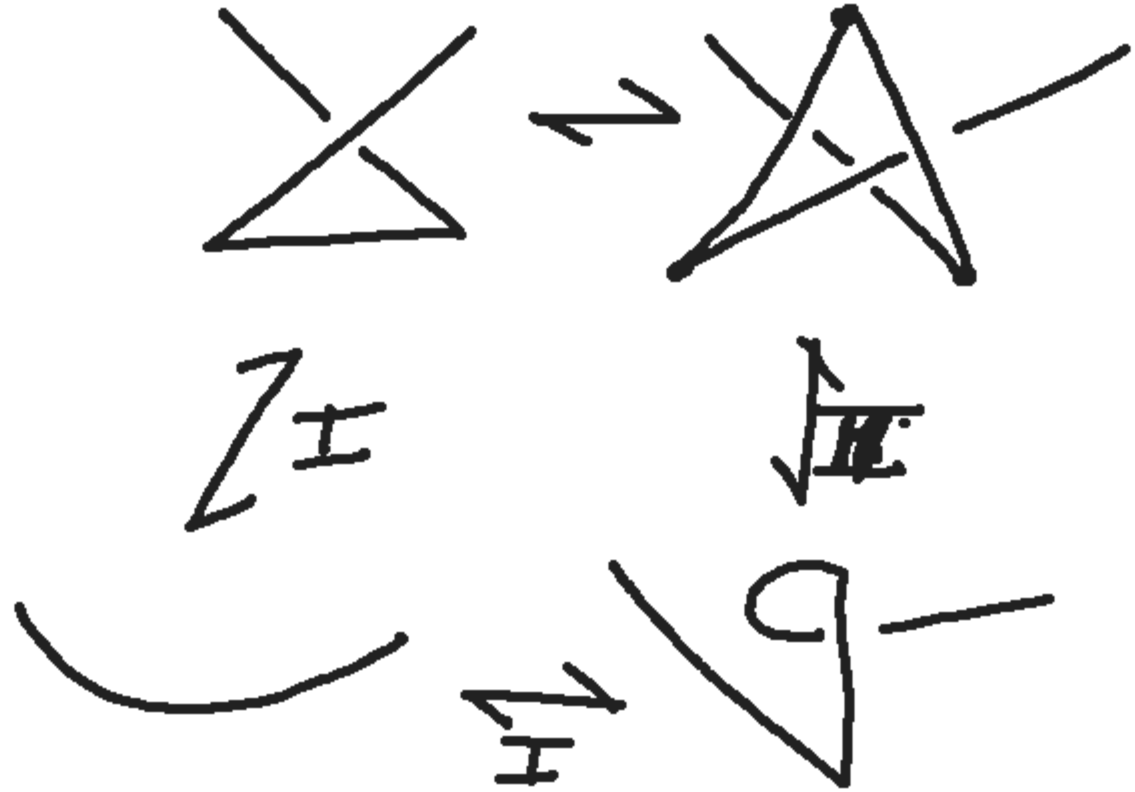
Smallest ones
- one line
- one crossing
- edge lines

end result

In the projections all move can be accomplished via





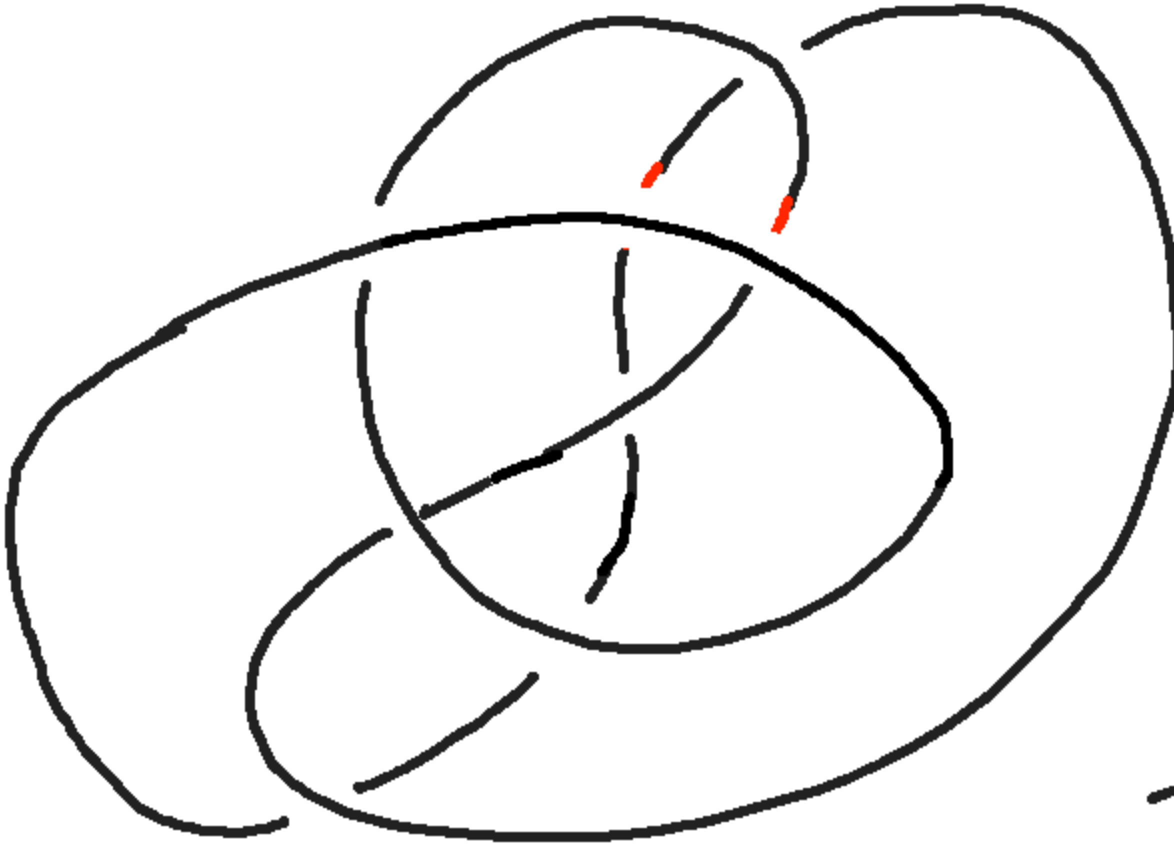


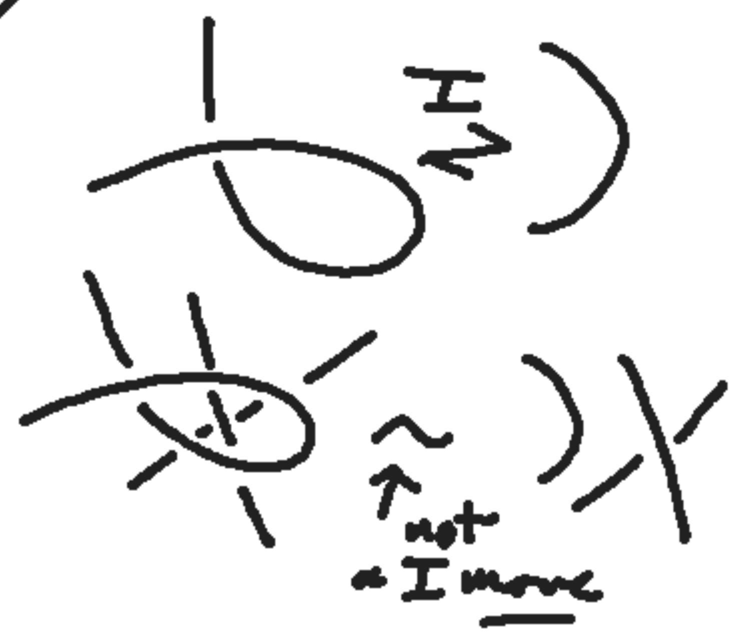
Thm (R, A, B)

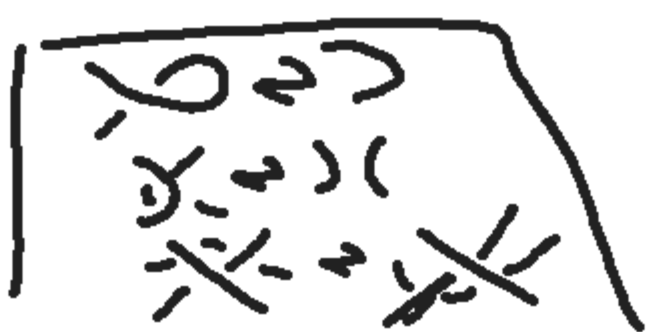
Two diags D, D' rep amb isotopic knots $\Longleftrightarrow$ $D \leftrightarrows \leftrightarrows \cdots \leftrightarrows D'$

moves $=$, I, II, III.

unknot this using R Moves.



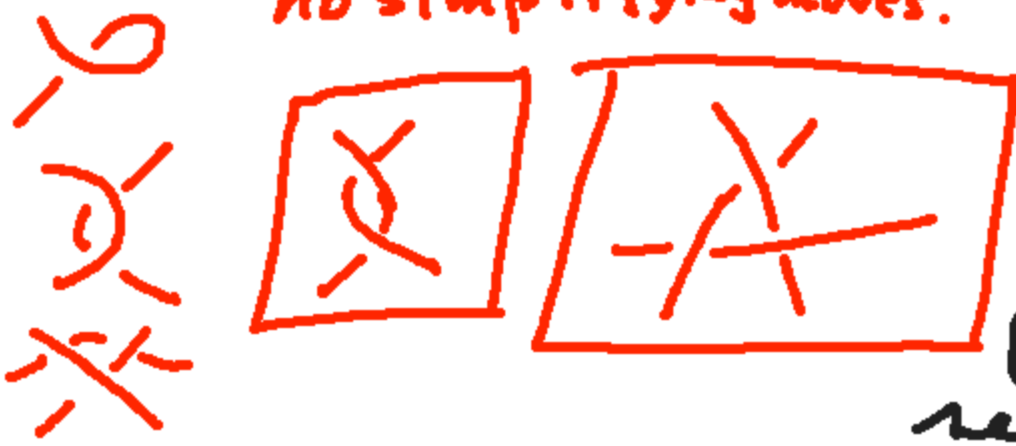




ambient isotopic

amb iso $\sim$

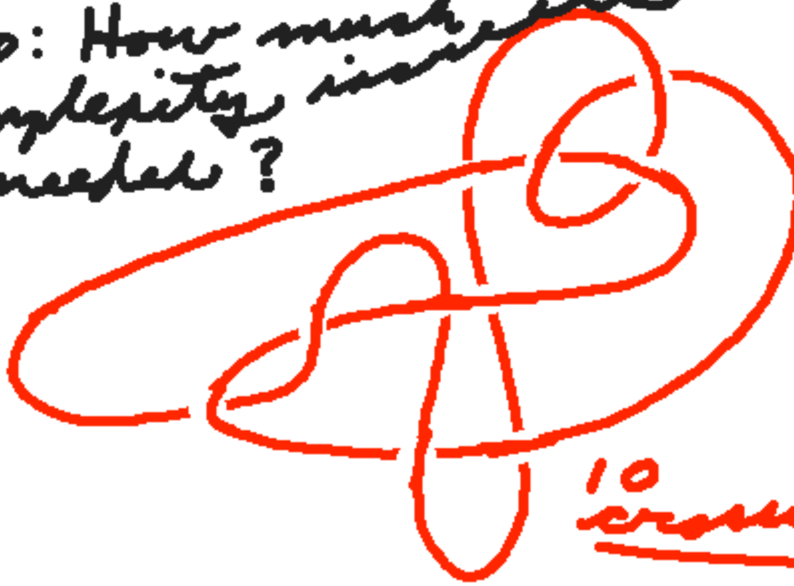
no simplifying moves.



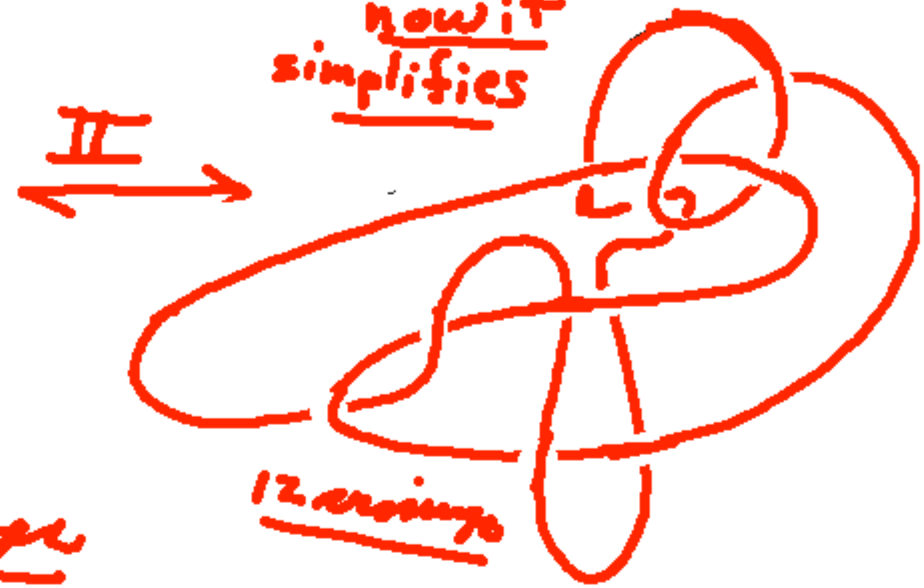
have to make diag move complex (# crossings) before it simplifies.

(Later we will see Dynnikov's moves ...)

Prob: How much extra complexity is needed?



now it simplifies



no simplifying moves.

have to make diag more complex (# crossings) before it simplifies.

(Later we will see Dynnikov's moves ...)

$\boxed{\underline{\text{link}}: \; S^1 \sqcup S^1 \sqcup \cdots \sqcup S^1 \hookrightarrow \mathbb{R}^3}$

- ambient isotopy of knots in $\mathbb{R}^3$ or S^3

$\Updownarrow$ $\boxed{\text{RAB Thm}}$

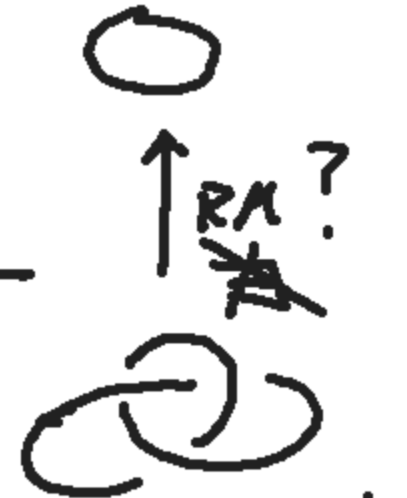
- diagram R-equiv of knot diags in plane

$\boxed{\text{we can study links the same way}}$

4 valent plane graphs

with extra (crossing) str

Next: Invariants of Knots + Links



knotted

- linking #
- fundamental group
- quandle → • colorings
- AlexanderPoly
- JonesPoly + Kauffman bracket
- virtual knot theory

Whitehead Link