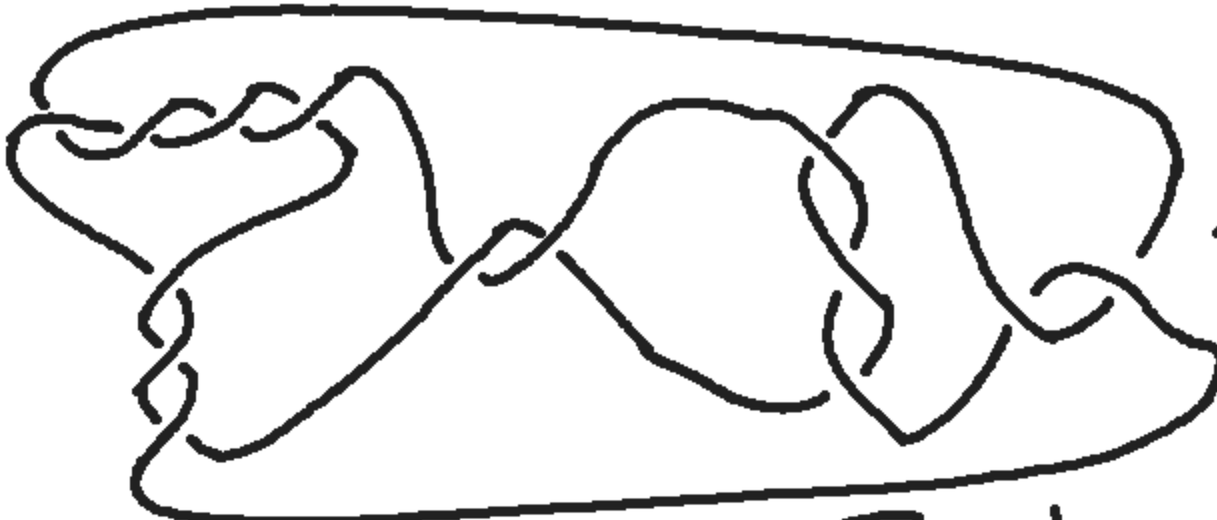
Knot Theory Problem Set

L.K. Fall 2020

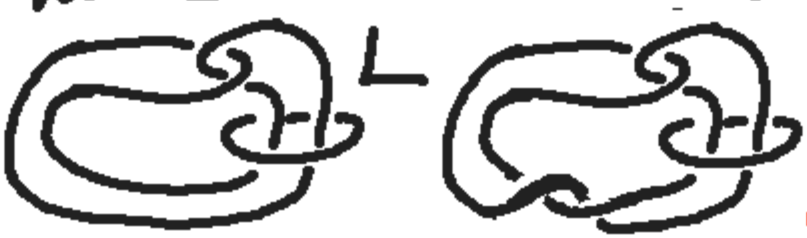
1.



K

Give a sequence of Reidemeister moves that show that K above is unknotted.

2. Prove that the two links below have homeomorphic complements.

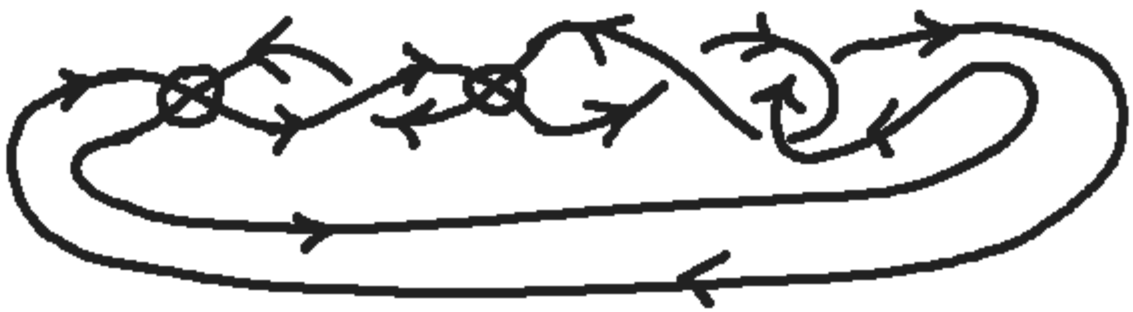


L L′

Prove that $L \not\sim L'$ by computing $\langle L \rangle$ and $\langle L' \rangle$ and normalizing them.

3. Give a proof, using 3-coloring, that the link B shown below is non-trivial.

B

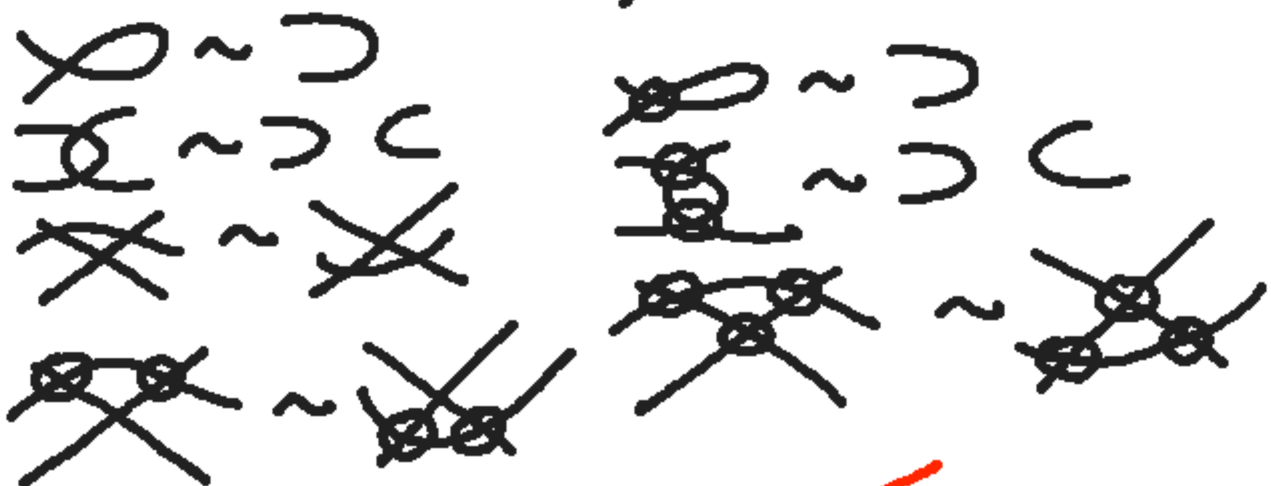
4. 

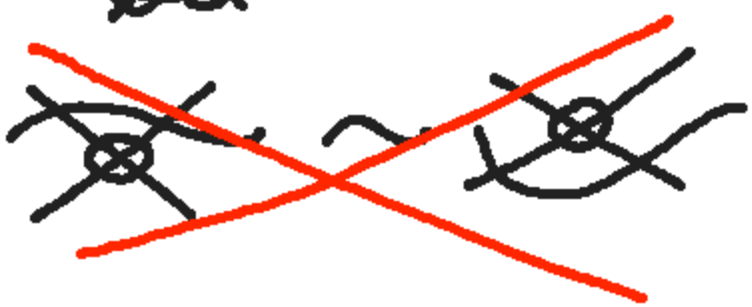
K

(a) Prove that K (as a virtual knot) has normalized bracket $f_K(A) = 1$. (We say K has unit Jones polynomial.)

(b) Prove that K is a non-classical, non-trivial knot by computing its affine index polynomial $P_K(t)$.

5. Recall that for <u>flat</u> <u>virtual</u> <u>knots</u> we have classical flat crossings  and virtual crossings  with the moves:

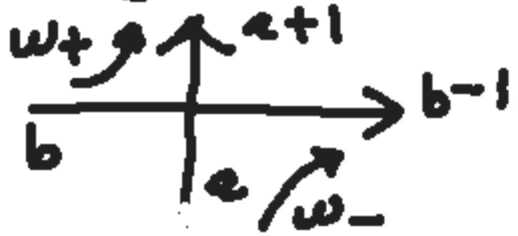


But  is <u>Forbidden</u>.

Recall also that we used flat diagrams to calculate the affine index polynomial $P_K(t)$.

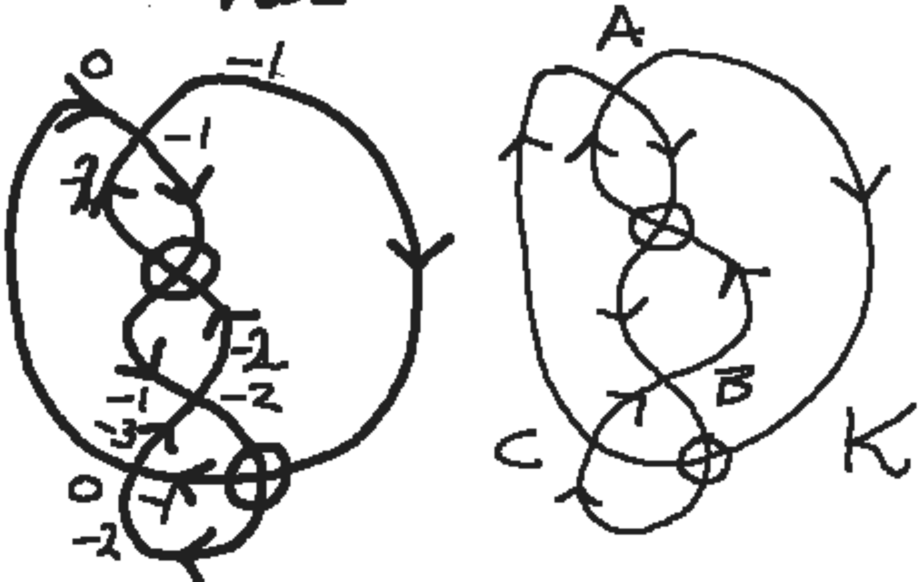
Defining the Q-Polynomial for Flat Virtual Knots.

Crossing weights are defined by integer labels.



$$\left.\begin{array}{l} \omega_+ = b - a - 1 \\ \omega_- = a - b + 1 = -\omega_+ \end{array}\right\}$$

e.g.



	ω_+	ω_-
A	1	−1
B	1	−1
C	−2	2

Define $Q_K(t)$ for flat diagrams

by the formula: $Q_K(t) = \sum_c t^{\omega_+(c)} - t^{\omega_-(c)}$.

(Thus above $Q_K(t) = 2(t - t^{-1}) + (t^{-2} - t^{2})$.)

(a) Prove that $Q_K(t)$ is invariant under the moves for flat virtual diagrams as we described them at the beginning of this problem.

(b) If K is a flat virtual diagram, let $c(K)$ denote the number of classical crossings $(\times)$ in the diagram K.

We see that

$$Q_K(t) = n_1(t^{k_1} - t^{-k_1}) + \cdots + n_\ell(t^{k_\ell} - t^{-k_\ell})$$

where $n_i > 0$ and $k_i \neq 0 \ \forall i$. Prove that

$$(n_1 + n_2 + \cdots + n_\ell) \leqslant c(K).$$

(c) Discuss (b) in relation to the example on the previous page.