

# Recombination of Vortex Loops in Helium and Theory of Quantum Turbulence

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- My talk will be very different from previous talks at this online conference. I had great doubts and shared these doubts with Professor Andrei Vesnin. Andrey, in turn, shared his doubts with Professor L. Kauffman .
- Reaction of Prof. Kaufman was about the following:
- I think that a good place to start would be the material about recombination of vortex loops in PHYSICAL REVIEW B 77, 214509 ,2008 "Kinetics of a network of vortex loops in He II and a theory of superfluid turbulence" by Sergey K. Nemirovskii\* This is very geometric and would be of interest to everyone in the seminar. **If he is willing to give more than one talk, I would very much like to hear about Quantum Turbulence.** It is great that Sergey Nemirovskii is willing to speak in our seminar. I am very much looking forward to his talks.

- Following Professor Kauffman's suggestion, I decided, instead of a separate talk, to combine the two talks today and present an extended Introduction, which will essentially be a small review on quantum turbulence. It seems to me that this topic will be (firstly) interesting for the audience, and (secondly) will allow me to smoothly move on to the main problem of the role of reconnections in the formation of quantum turbulence
- **PART I . Quantum Turbulence (overview).**

**First observation of the lambda transition.  
Bubbly and calm boiling of helium.  $T = 2.17$  K**





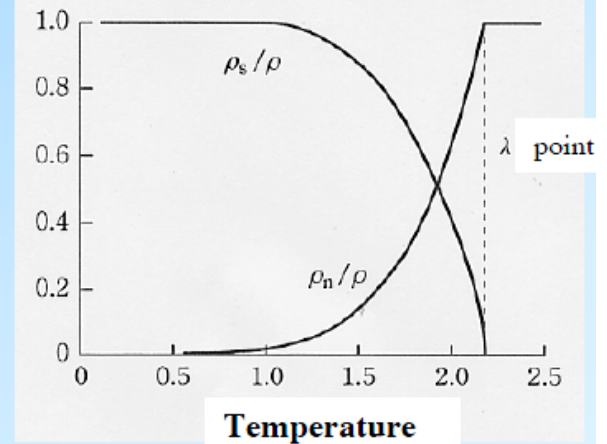
# Two-fluid Landau model

From the point of view of hydrodynamics, the new phase of helium, the so-called helium II (He II), can be represented as a mixture of two liquids

## The two-fluid model

The system is a mixture of inviscid superfluid and viscous normal fluid.

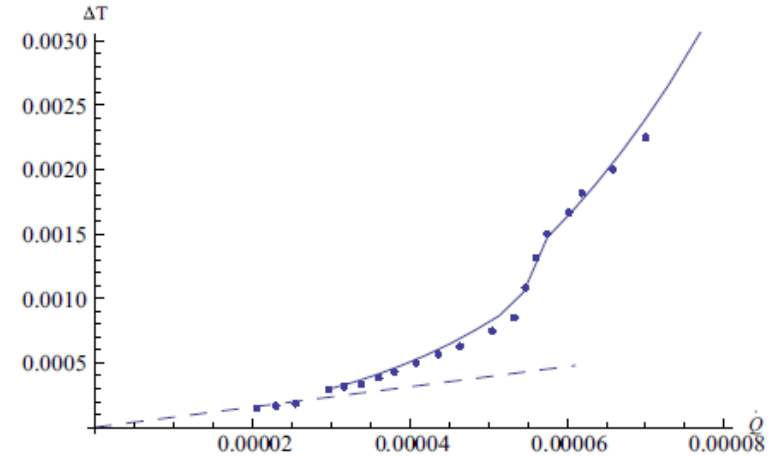
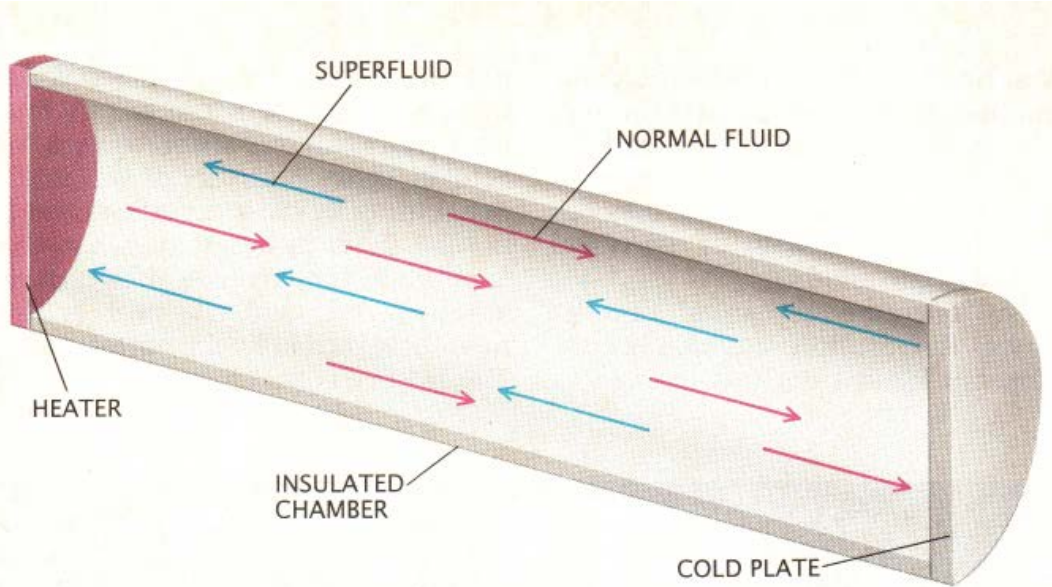
$$\rho = \rho_s + \rho_n \quad \mathbf{j} = \rho_s \mathbf{v}_s + \rho_n \mathbf{v}_n$$



	density	velocity	viscosity	entropy
Superfluid	$\rho_s(T)$	$\mathbf{v}_s(\mathbf{r})$	0	0
Normal fluid	$\rho_n(T)$	$\mathbf{v}_n(\mathbf{r})$	$\eta_n(T)$	$s_n(T)$

$$\nabla \times \mathbf{v}_s = 0$$

# Counterflow. High thermal conductivity.



**TWO-FLUID MODEL** explains many properties of superfluid helium. According to this model, a sample of superfluid helium is made up of two interpenetrating fluids: a superfluid (*blue*), which flows without friction and, in one sense, has a temperature of absolute zero, and a normal fluid (*red*), which flows with normal friction and carries all the heat in the sample. A heater at one end of a channel of superfluid helium causes a counterflow: normal fluid “created” at the heater flows toward the other end of the channel while superfluid flows back in the opposite direction.

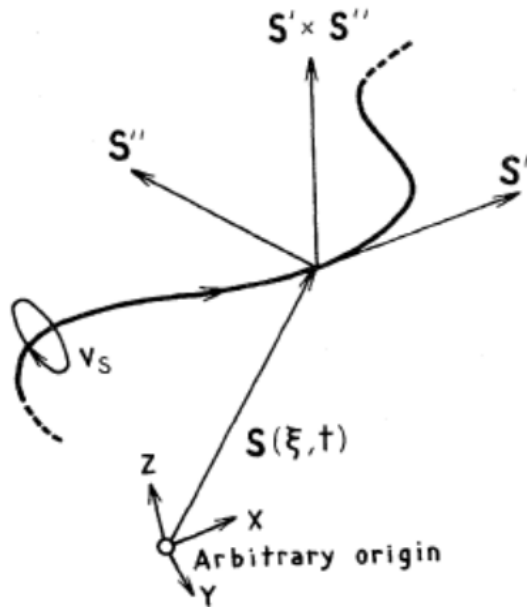
$$\nabla T \propto \dot{Q}$$

$$\nabla T \propto \dot{Q}^3$$

# Superfluid (quantum) turbulence in He II. Vortex tangle.



# Vortex filaments in He II. Deterministic dynamics



$$\nabla \times \mathbf{v}_s = 0$$

$$\oint \mathbf{v}_s \cdot d\mathbf{l} = n\kappa.$$

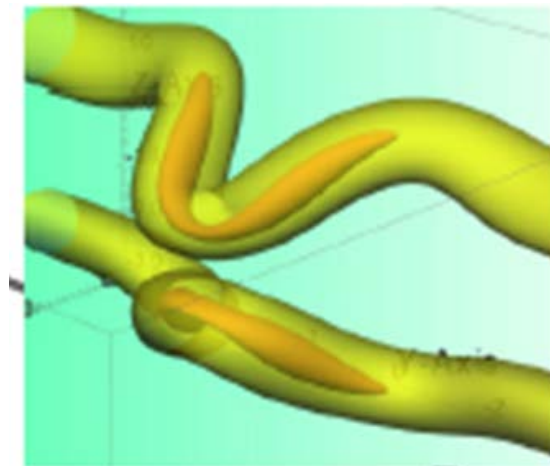
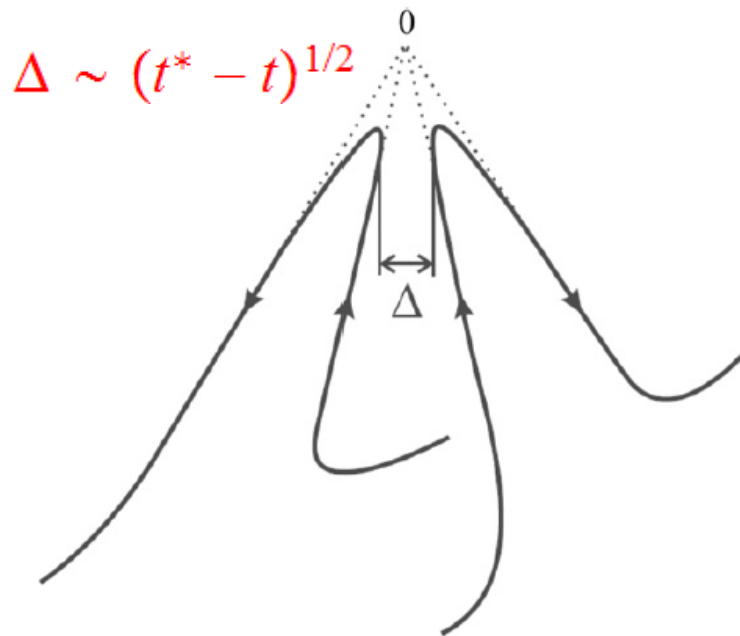
$$\kappa = 2\pi\hbar/m_{He} = 9.97 \cdot 10^{-4} \text{ cm}^2/\text{s}$$

$$\mathbf{v}_s(\mathbf{r}) = \left(0, \frac{\kappa}{2\pi r}, 0\right)$$

$$\dot{\mathbf{s}} = \dot{\mathbf{s}}_f + \alpha \mathbf{s}' \times (\mathbf{v}_n - \dot{\mathbf{s}}_f) - \alpha' \mathbf{s}' \times \mathbf{s}' \times (\mathbf{v}_n - \dot{\mathbf{s}}_f)$$

$$\dot{\mathbf{s}}_i(\xi, t) = \frac{\kappa}{4\pi} \int \frac{[\mathbf{s}(\xi', t) - \mathbf{s}(\xi, t)] \times \mathbf{s}'_{\xi'}}{|\mathbf{s}(\xi', t) - \mathbf{s}(\xi, t)|^3} d\xi'$$

# Reconnection of lines



What is it for??



# What is it for??

- Interest in quantum turbulence is motivated by several things. First of all, quantum turbulence as a part of the theory of superfluidity is closely connected with other problems of the general theory of quantum fluids.



# What is it for??

- One more, extremely important, line of interest in quantum turbulence, currently being intensively discussed, is the hope that the use of the theory of stochastic vortex lines will help to clarify the perennial problem of classical turbulence (or at least to explain some key features, like Kolmogorov spectra, intermittency etc.).



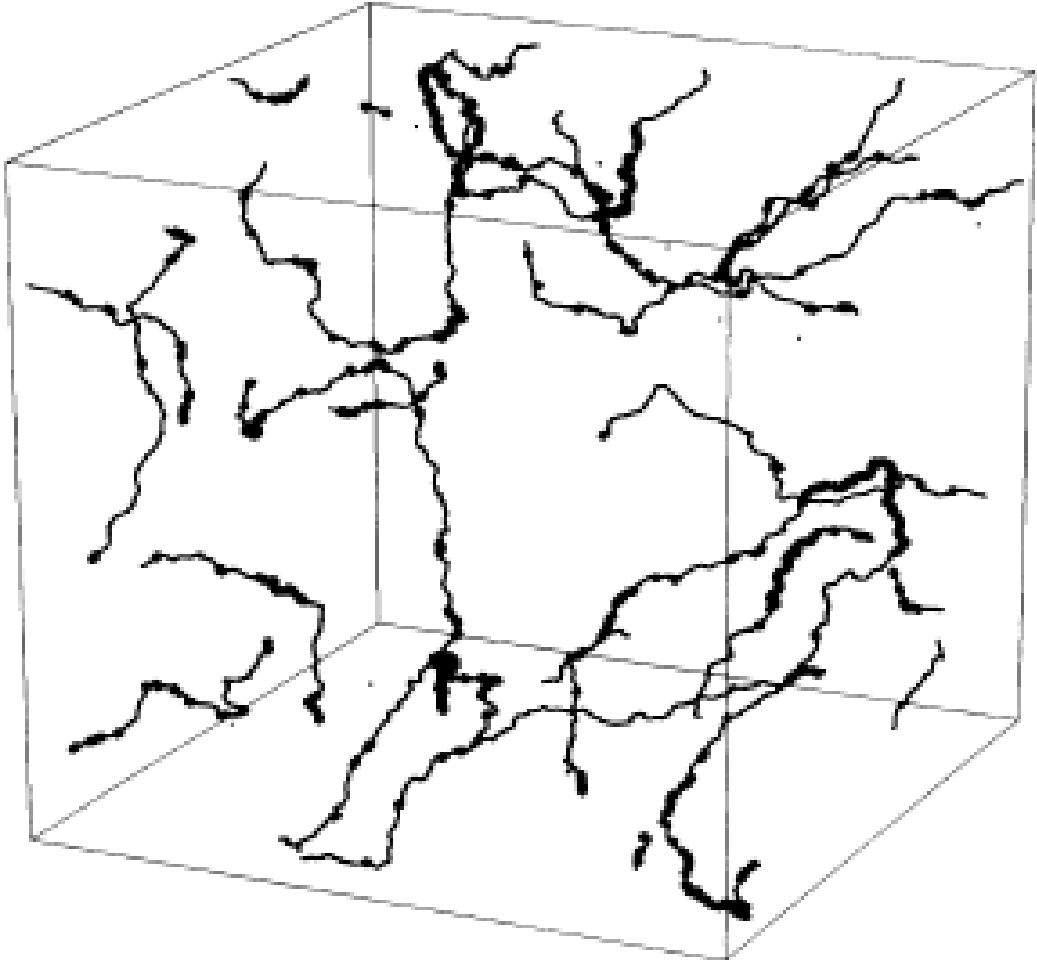
# What is it for??

- One more justification for the interest in quantum turbulence, attractive for theoreticians, is that the theory of superfluid turbulence is an elegant and challenging statistical problem used for the study of the dynamics of a chaotic set of string-like objects, with nonlinear and nonlocal interactions plus reconnections resulting in the fusion or splitting of vortex loops. The latter feature allows one to classify quantum turbulence as a variant of string field theory.

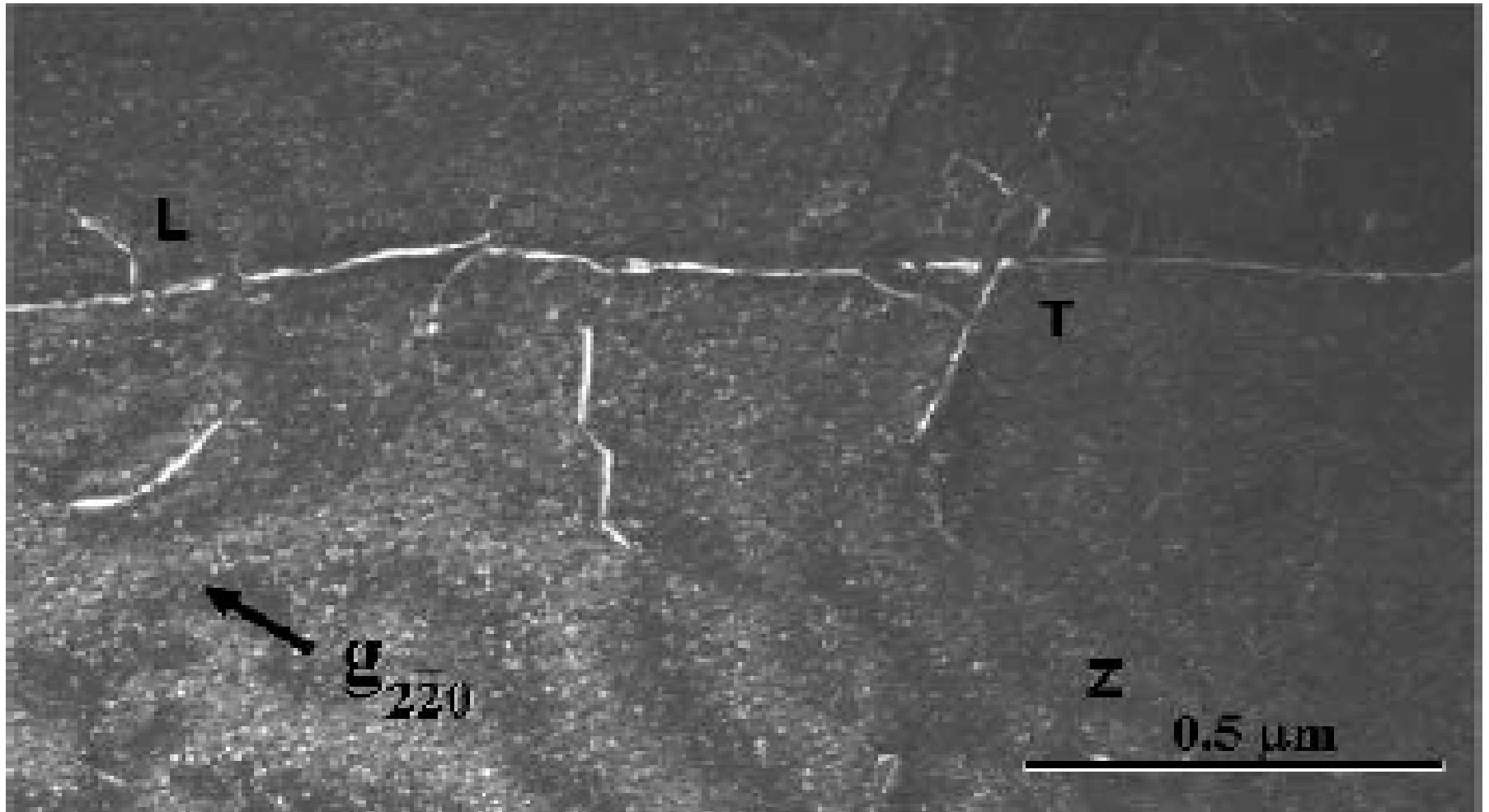
# What is it for??

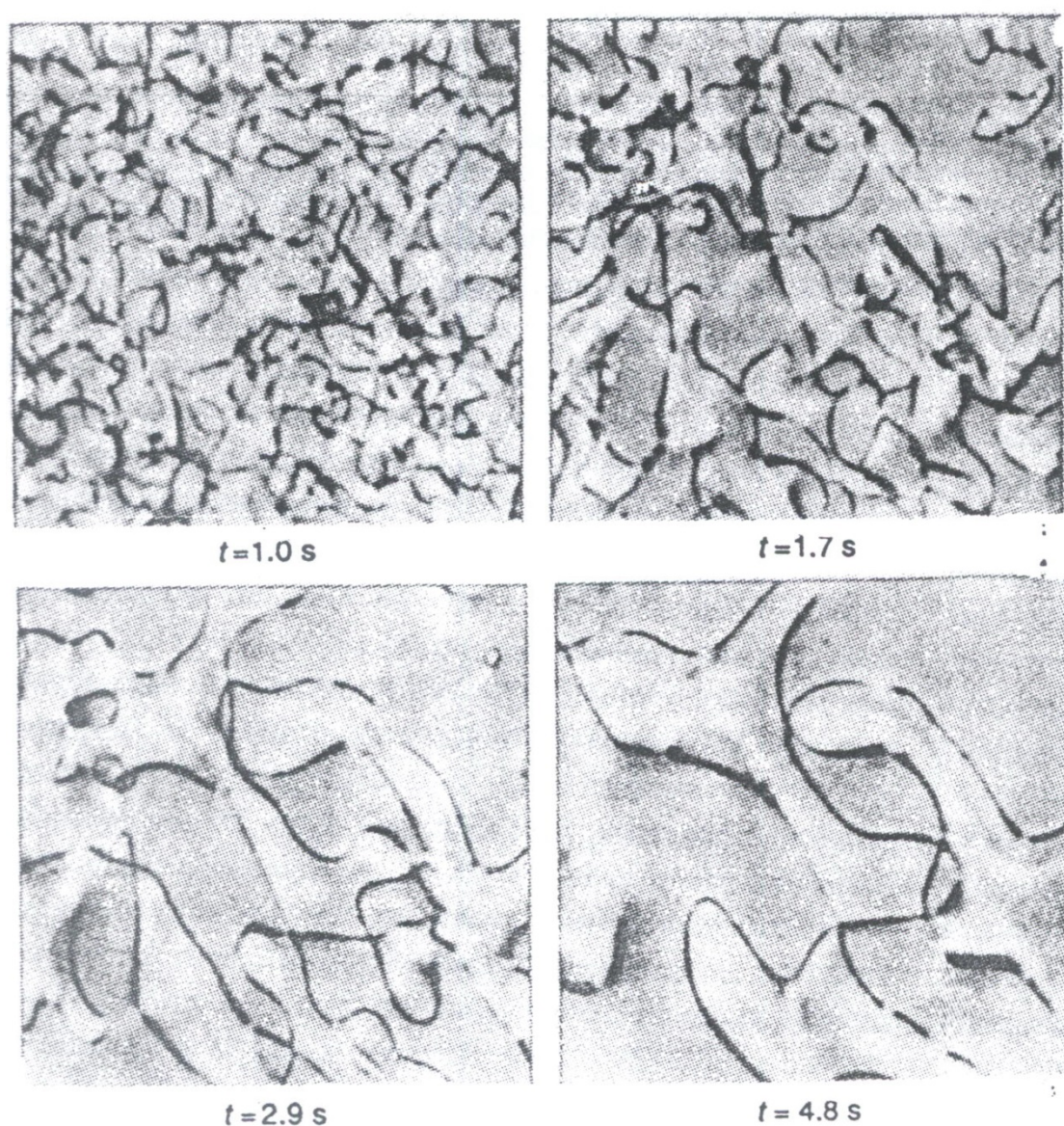
- Besides the great importance of superfluid turbulence in the above-mentioned cases, we would like to point out that the theory of the stochastic vortex tangle in quantum fluids is of great interest and importance from the point of view of general physics. This view is justified by the existence in many physical fields of similar systems of highly disordered sets of one-dimensional (1D) singularities.

Network of cosmic strings (D. Bennet, F. Bouchet, 1989 )



# Dislocations in solids, C. Deeb et al. (2004)





Tangle of string-like defects in nematic liquid crystal

I. Chuang, R. Durrer,  
N. Turok, B. Yorke,  
("Science", 1991)

**Fig. 4.** A coarsening sequence showing the strings visible in our 230- $\mu\text{m}$ -thick pressure cell containing K15 nematic liquid crystal, at  $t = 1.0, 1.7, 2.9,$  and  $4.8$  seconds after a pressure jump of  $\Delta P = 4.7$  MPa from an initially isotropic state in equilibrium at approximately  $33^\circ\text{C}$  and  $3.6$  MPa. The evolution of the string network shows self-similar or "scaling" behavior. Each picture shows a region  $360$   $\mu\text{m}$  in width.

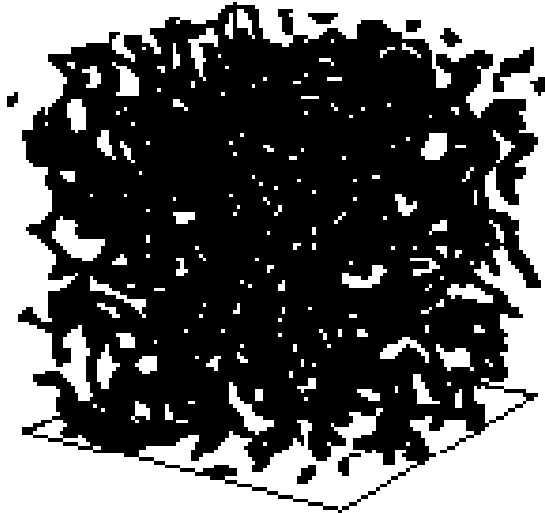


**Natural light fields are threaded by lines of darkness. They are optical vortices that extend as lines throughout the volume of the field.**

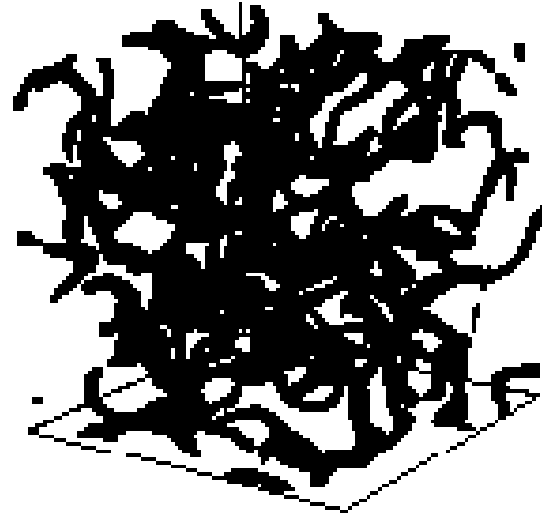


# Topological defects in Bose Gas (N. Berloff, B. Svistunov, 2001)

$t = 1000$



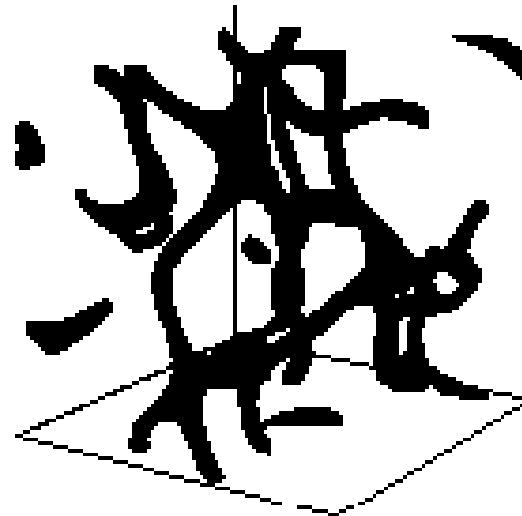
$t = 2000$



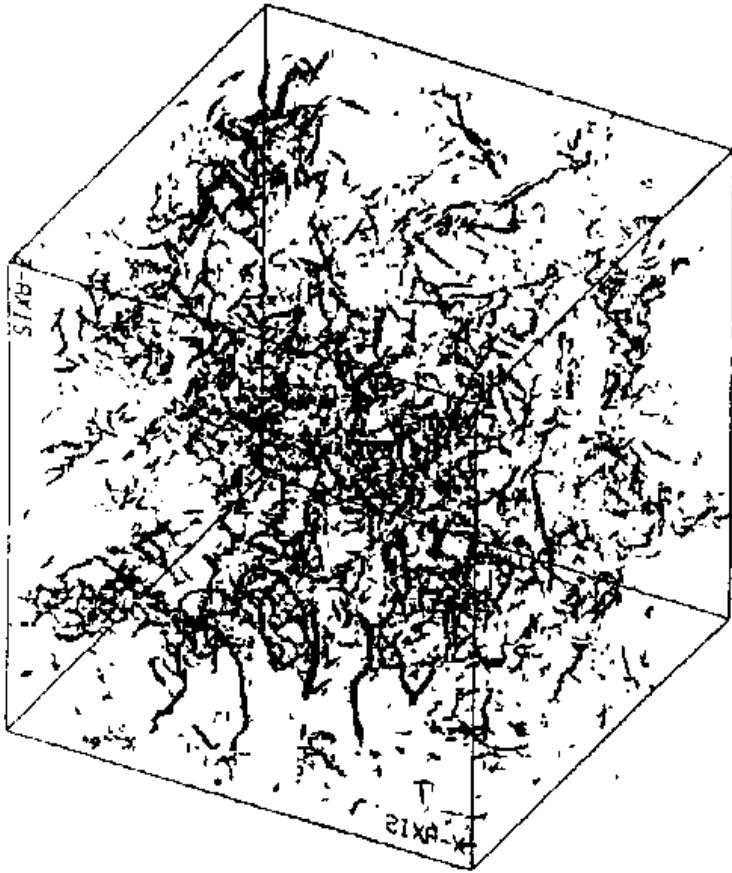
$t = 3000$



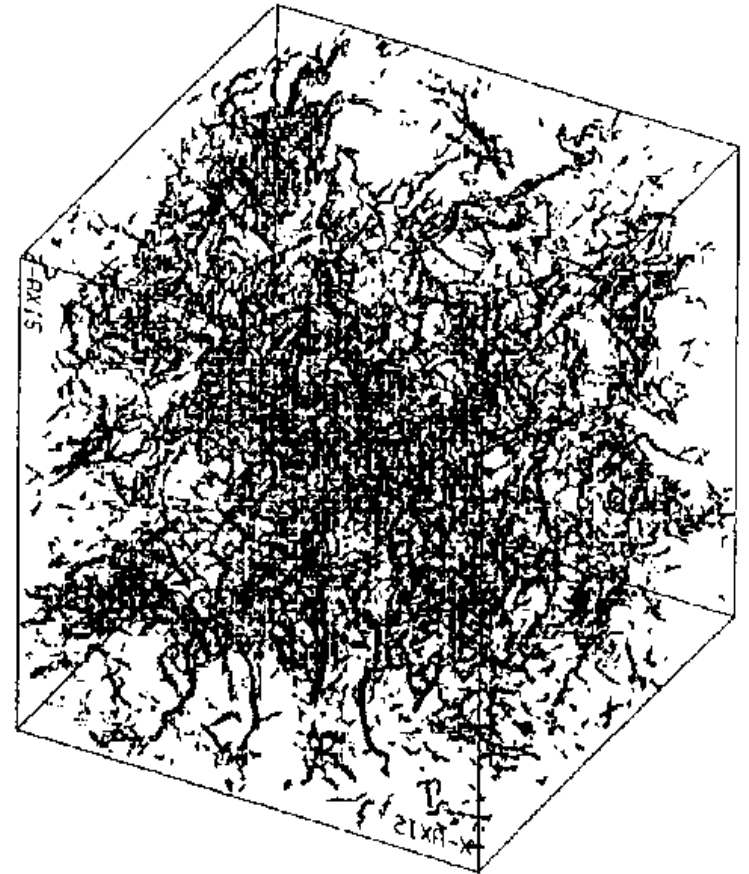
$t = 5000$



Tangle of vortex filaments obtained in turbulent flow at moderately high Reynolds (Vincent and Meneguzzi 1991).



(a)

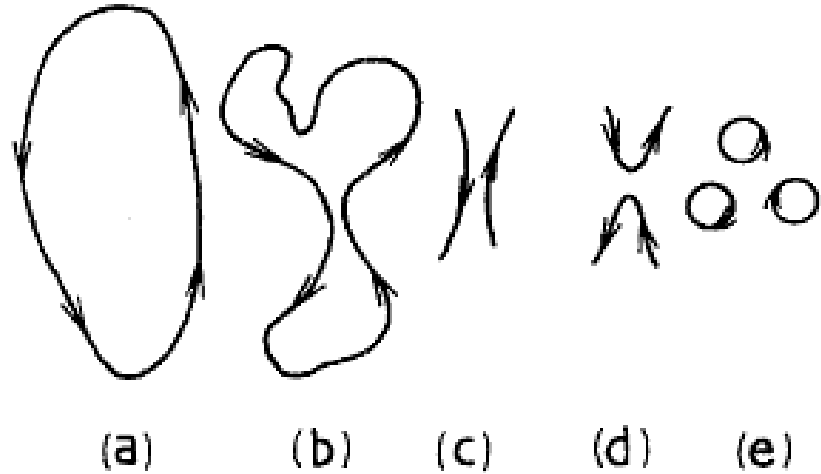
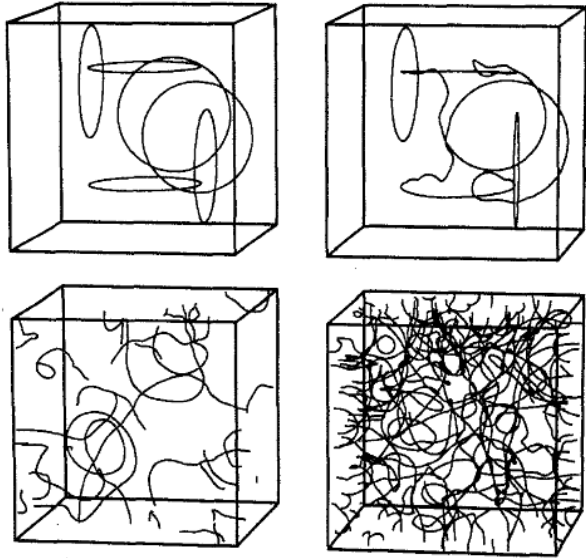


(b)



**About the main trends and key results.**

# Feynman's qualitative model and Vinen phenomenological theory



$$\frac{dL}{dt} = \alpha_V |\mathbf{v}_{ns}| L^{3/2} - \beta_V L^2 .$$

$$\mathcal{L} = \left( \frac{\alpha}{\beta} \right)^2 |\mathbf{v}_{ns}|^2 = (\gamma(T))^2 |\mathbf{v}_{ns}|^2$$

$$R \sim \delta = \mathcal{L}^{-1/2}$$

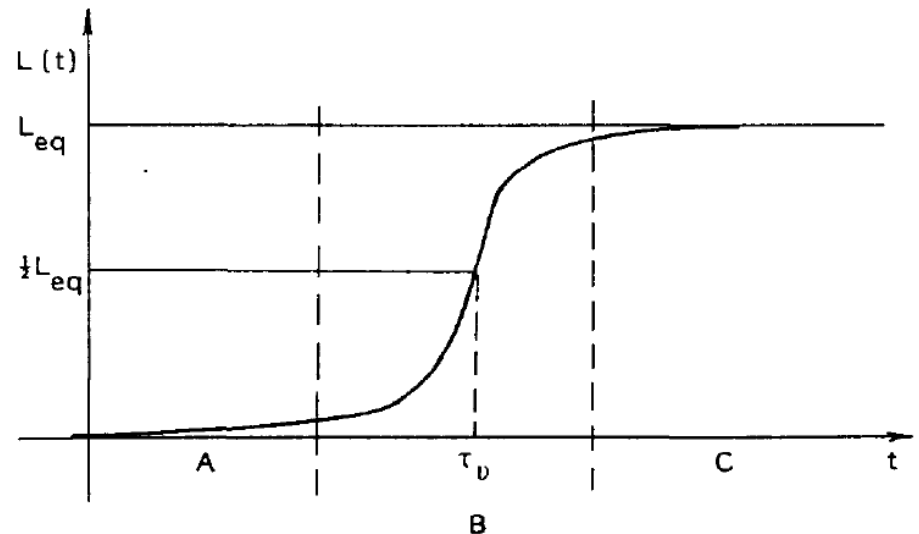
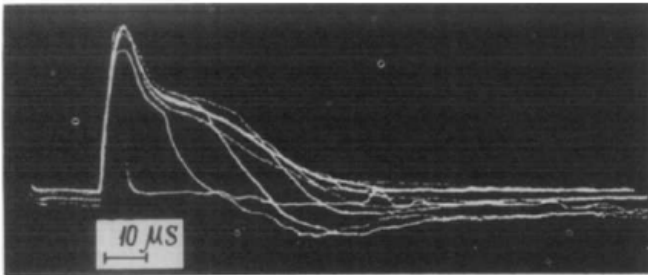
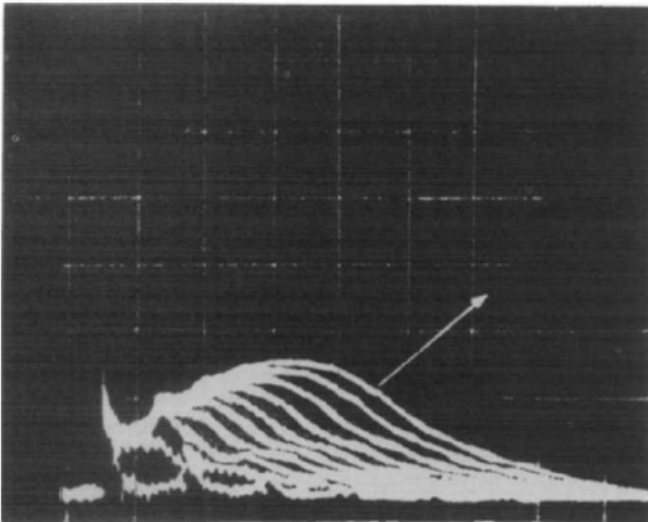


Figure 9 Possible shape of the function  $L(t)$

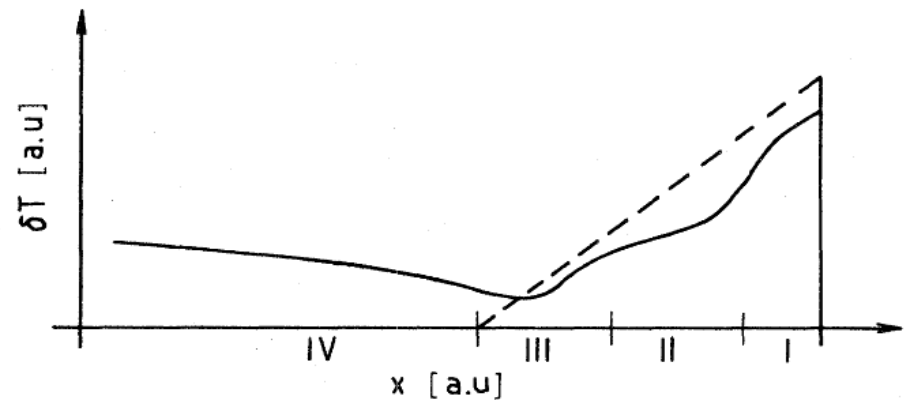
# Dynamics of heat pulses



**Figure 3** Appearance of the limiting profile of a second sound pulse as  $t_s$  increases:  $t_s = 3, 18, 30, 40, 50, 60,$  and  $70 \mu\text{s}$ .  $W = 73.5 \text{ W cm}^{-2}$ ,  $T = 1.884 \text{ K}^{23}$

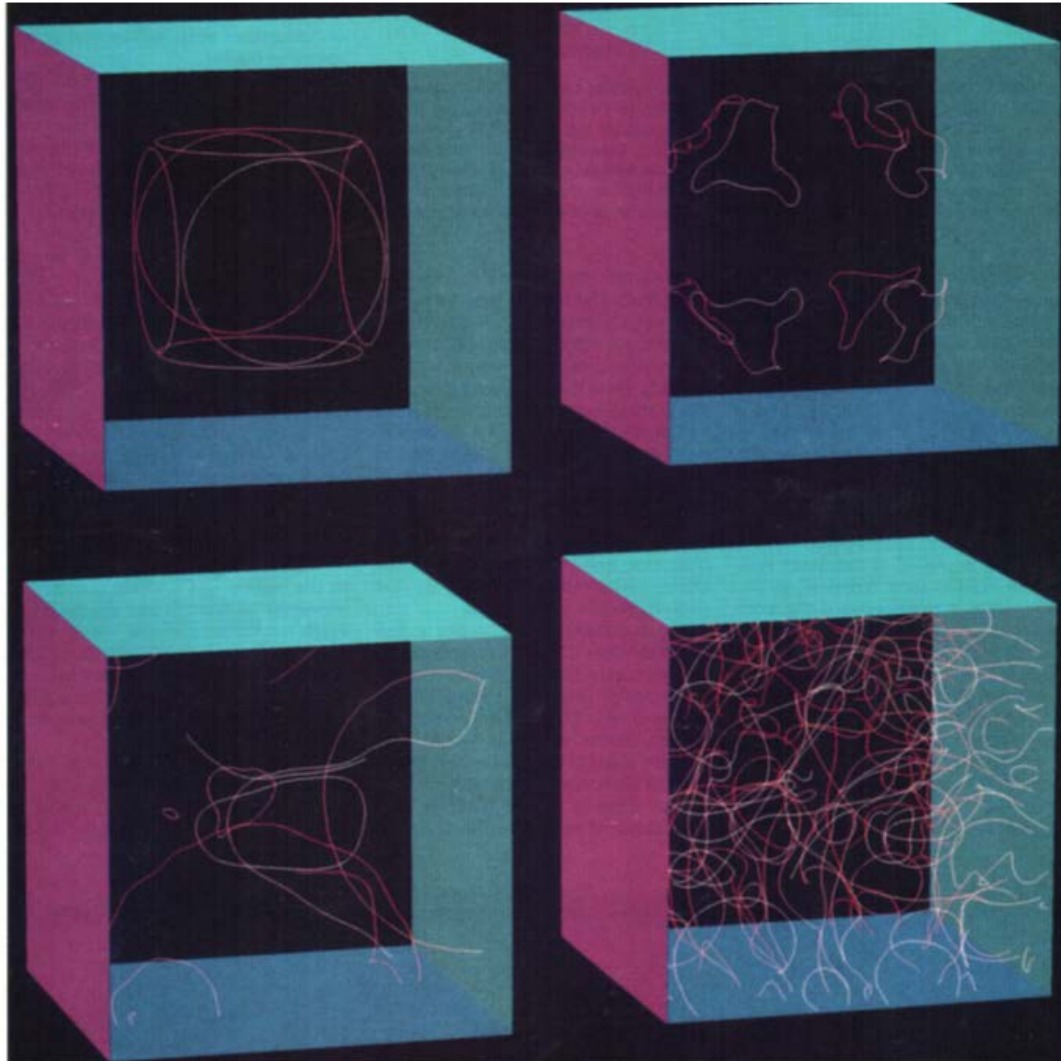


**Figure 4** Secondary temperature fronts produced by heat pulses in the presence of the vortex tangle. Oscilloscope traces show temperature *versus* time ( $10 \text{ ms/div.}$ ). Arrow indicates how temperature profiles progressively develop as  $W$  increases. At the very beginning of the oscilloscope traces, there are second sound shock waves<sup>18</sup>



**FIG. 28.** Schematic of the distortion of the temperature pulse due to the interaction with its "own" vortex lines (Nemirovskii and Schmidt, 1990, Fig. 7). The dashed line represents the vortexless case when the pulse should be a "Burgers" triangle.

# 1<sup>st</sup> numerical simulations (K.W. Schwarz 1988)



# Развитие квантовой турбулентности



# Динамика Бозе-конденсата, Питаевский, Гросс.

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{\hbar^2}{2m} \Delta \psi + U_0 (|\psi|^2 - n) \psi$$

$$\psi(\mathbf{r}, t) = \sqrt{\rho(\mathbf{r}, t)/m} \exp(i\phi(\mathbf{r}, t))$$

$$\rho(\mathbf{r}, t) = m|\psi|^2, \quad \mathbf{v} = \frac{i\hbar}{2m} (\psi \nabla \psi^* - \psi^* \nabla \psi) = \frac{\hbar}{m} \nabla \phi.$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0,$$

$$\rho \left( \frac{\partial \mathbf{v}_j}{\partial t} + \mathbf{v}_k \frac{\partial \mathbf{v}_j}{\partial r_k} \right) = -\frac{\partial p}{\partial r_j} + \frac{\partial \Sigma_{jk}}{\partial r_k}.$$

$$p = \frac{U_0}{2m^2} \rho^2, \quad \Sigma_{jk} = \left( \frac{\hbar}{2m} \right)^2 \rho \frac{\partial \ln \rho}{\partial r_j \partial r_k}, \quad c = \sqrt{\frac{U_0 n}{m}}.$$

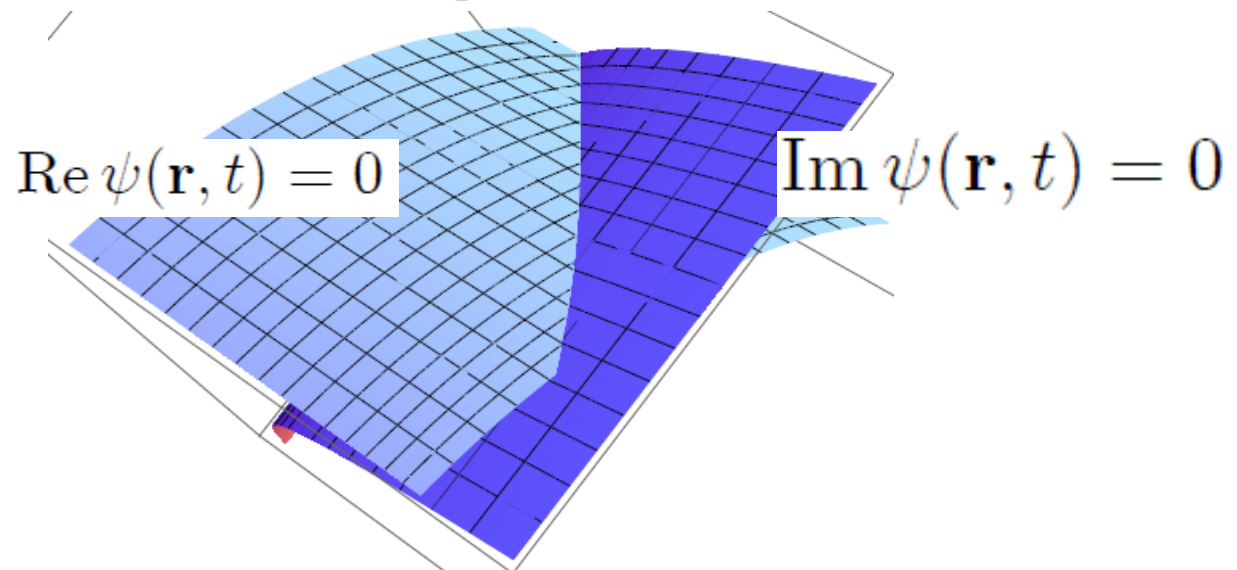
# Нуль параметра порядка (топологический дефект)

$$\psi(\mathbf{r}, t) = 0, \quad \psi(\mathbf{r}, t) = az + \dots = a(x + iy) + \dots$$

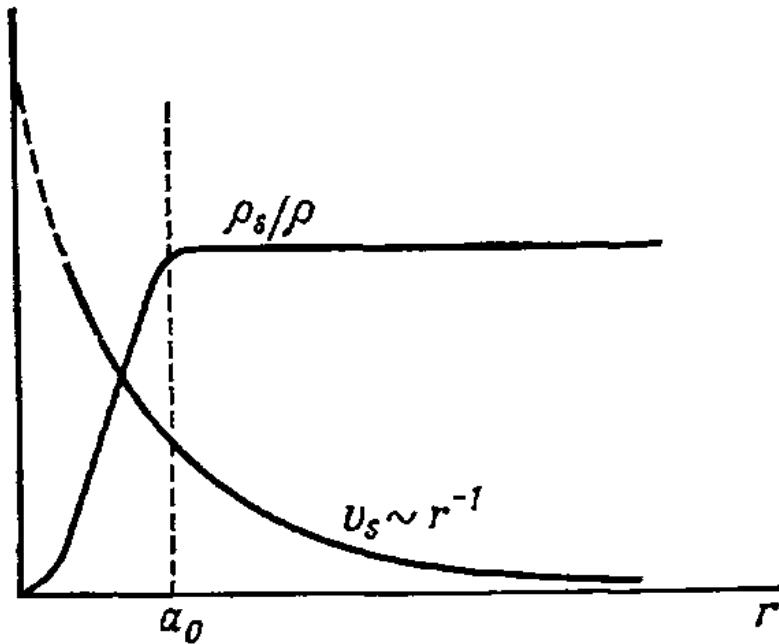
$$\psi(\mathbf{r}, t) = \sqrt{x^2 + y^2} \exp\left(i \arctan\left(\frac{y}{x}\right)\right) + \dots$$

$$\nabla \cdot \phi(\mathbf{r}, t) = \nabla \cdot \arctan\left(\frac{y}{x}\right) = \frac{\hbar}{m} \left( -\frac{y}{x^2 + y^2}, \frac{x}{x^2 + y^2} \right)$$

$$\mathbf{v} = \left(0, \frac{\hbar}{m} \frac{1}{r}, 0\right), \quad \oint \mathbf{v} \cdot d\mathbf{l} = \frac{2\pi\hbar}{m} = \kappa$$



# Структура квантового вихря в ВЕС

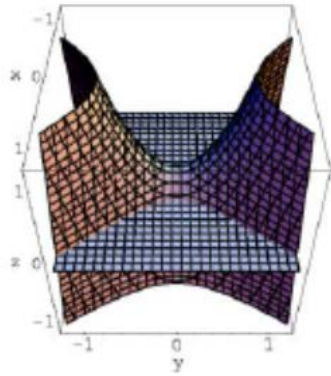


$$\psi = \sqrt{n} \exp(i\varphi) f(r/a_0),$$

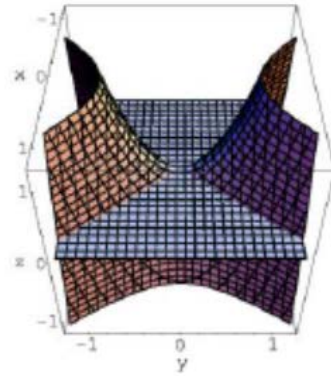
$$a_0 = \hbar / \sqrt{2mnU_0},$$



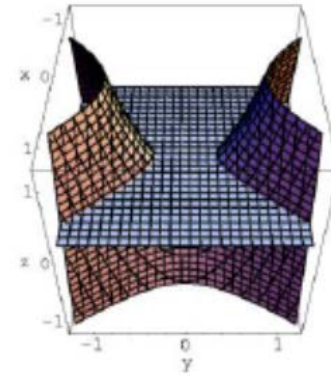
# Illustration to reconnection



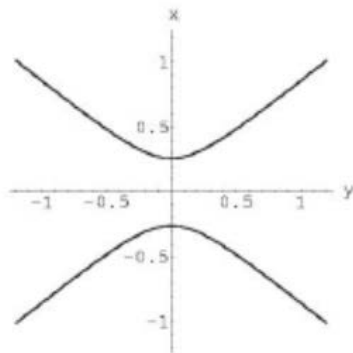
(a)  $t = -0.1$



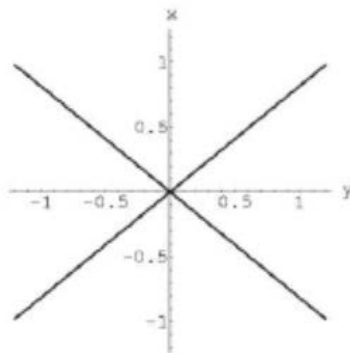
(b)  $t = 0.0$



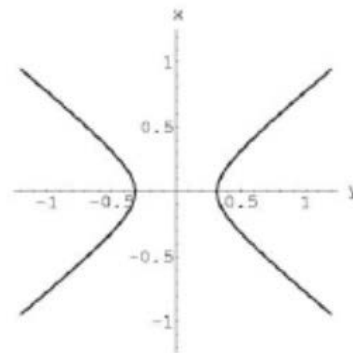
(c)  $t = 0.1$



(d)  $t = -0.1$

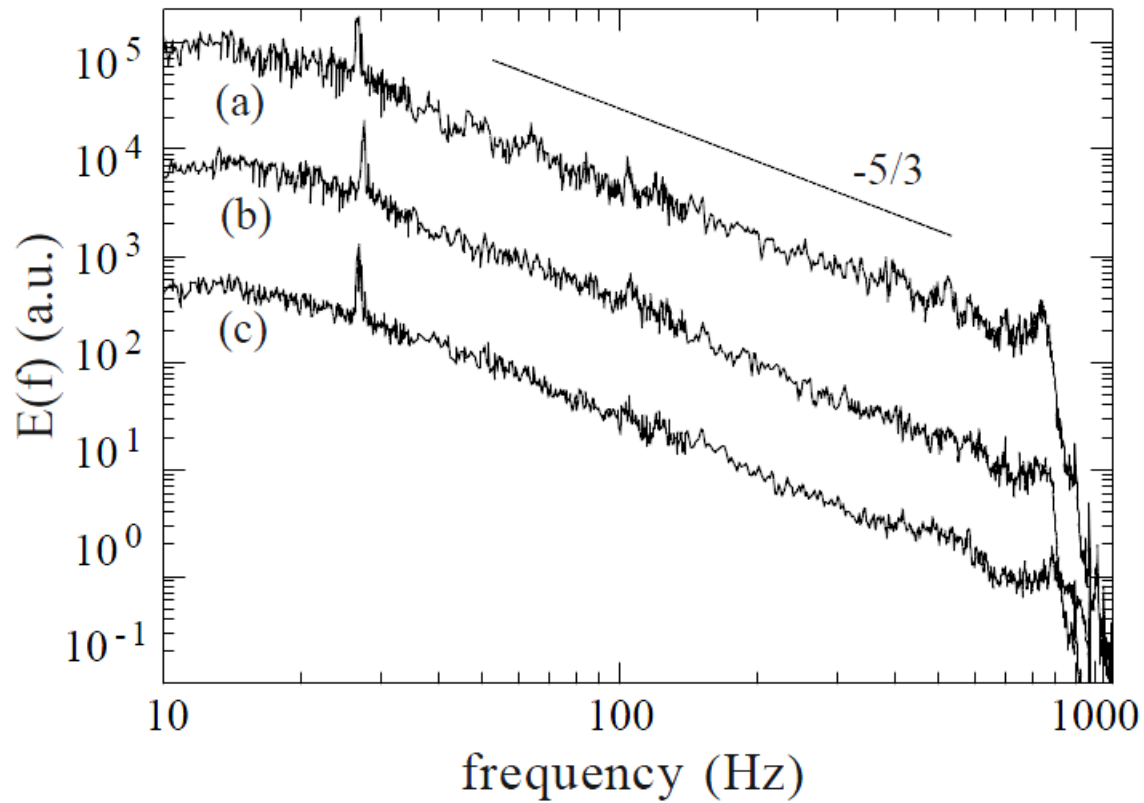


(e)  $t = 0.0$



(f)  $t = 0.1$

# Квазиклассическая турбулентность в сверхтекучих в жидкостях



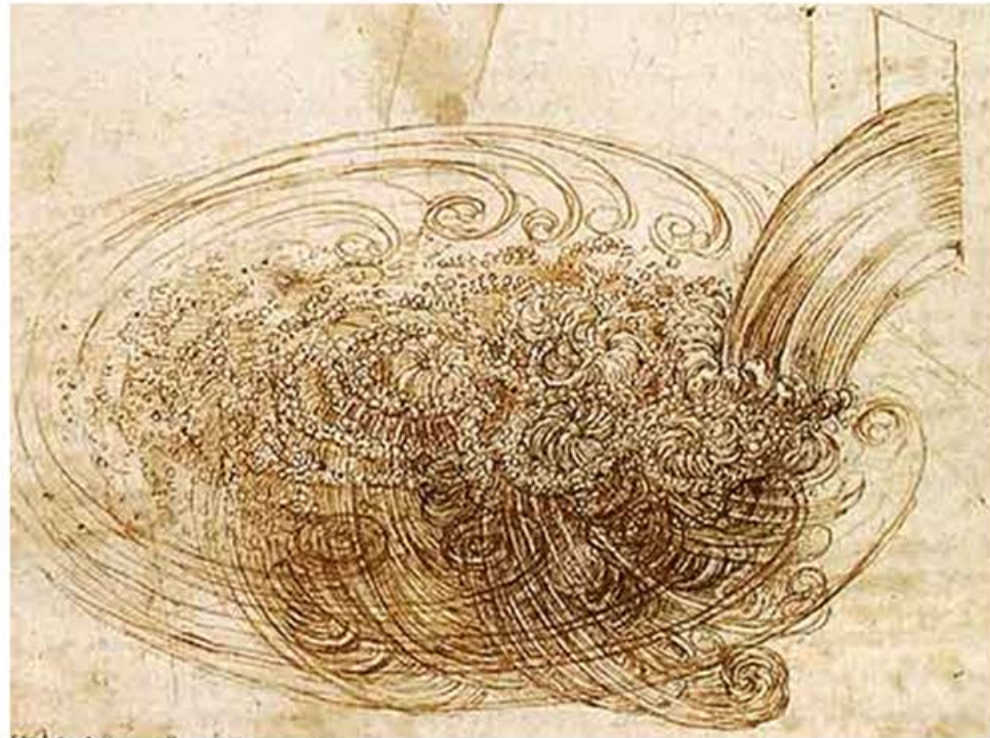
# Filamentary structure of classical turbulence.

## Modeling by vortex filaments

- As absolutely alternative way to resolve this problem is to treat turbulent features as consequence of dynamics of vortex filaments



Leonardo Da Vinci  
(1452-1519)



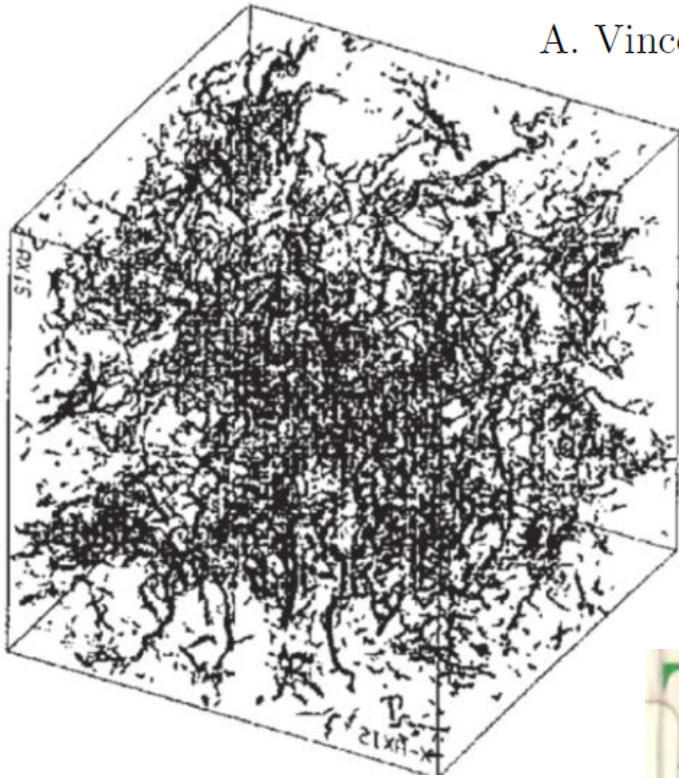
Da Vinci observed turbulent flow in water and found that turbulence consisted of many vortices.

From book by Frisch (1995)

# Filamentary structure of classical turbulence

Idea of modeling turbulence by discrete vortices on experimental and numerical evidences of that the developed turbulence has the vortex filamentary structure.

A. Vincent and M. Meneguzzi, *Journal of Fluid Mechanics* **225**, 1 (1991).



***8.9.2 Statistical signature of vortex filaments: dog or tail?***



# technical applications



Figure 2.9: View of CERN, Lake Geneva and the Alps. The circle denotes the location of the LHC.

the magnets of the LHC are cooled  
down to 1.9K with superfluid helium





Figure 2.10: The IRAS satellite.

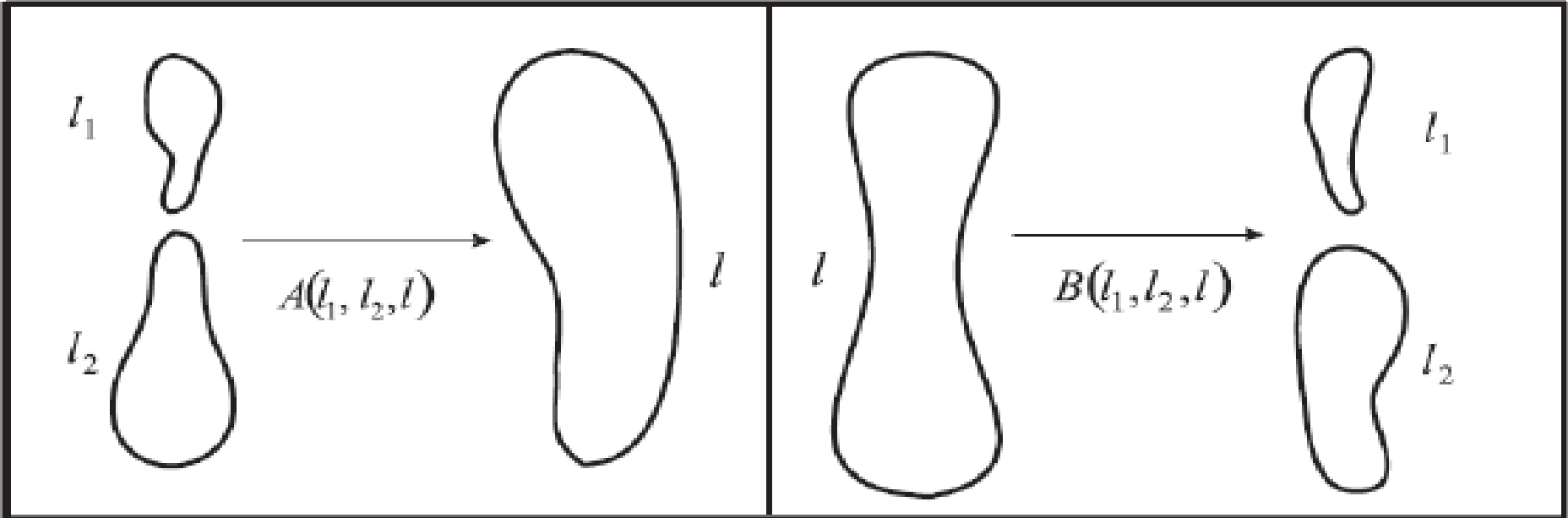
- **PART II . Recombination of Vortex Loops  
in Helium and Theory of Quantum Turbulence**



In general, the dynamics of the vortex tangle consists of two main ingredients.

The first one (deterministic) is the motion of the elements of lines, due to equations of the motion (Biot-Savart law, mutual friction etc.).

The second one is the (random) collisions (and merging), or self-intersections (and splitting) of the vortex loops.



Up to now the numerical results remain the main source of information about this process. The scarcity of analytic investigations is related to the incredible complexity of the problem. Indeed we have to deal with a set of objects which do not have a fixed number of elements, they can be born and die. Thus, some analog of the secondary quantization method is required with the difference that the objects (vortex loops) themselves possess an infinite number of degree of freedom with very involved dynamics. Clearly this problem can hardly be resolved in the nearest future. Some approach crucially reducing a number of degree of freedom is required.

# Recombination

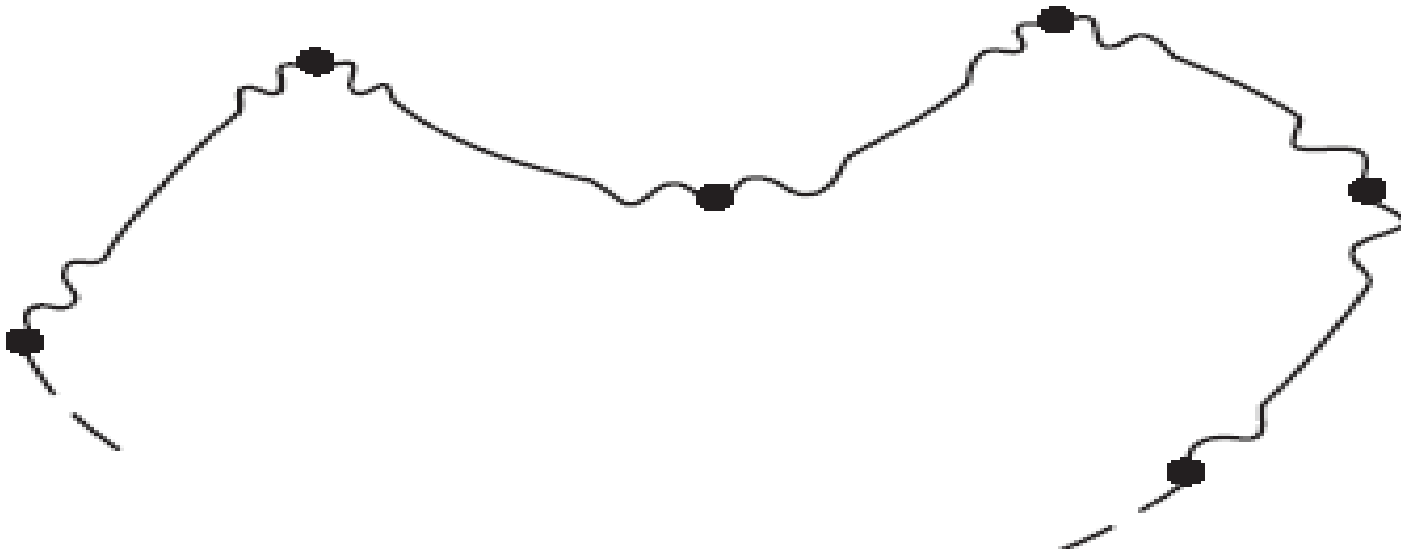
The full rate of reconnection  $\dot{N}_{rec}$  ( number of events per one second per unit volume) as a function of the vortex line density  $\mathcal{L}$  is estimated as

$$\dot{N}_{rec} \sim \kappa \mathcal{L}^{5/2}$$

where  $\kappa$  is quantum of circulation. Let us take for instance some typical experiments on superfluid turbulence, with the counterflowing velocity of order of 1cm/s. Under this conditions the value of the vortex line density  $\mathcal{L}$  is about  $\mathcal{L} \approx 10^4$  1/cm<sup>2</sup>. Then the full rate of reconnection  $\dot{N}_{rec}$  is about 10<sup>7</sup> collisions per second, or 10<sup>3</sup> collisions per each cm of line. Let us take a loop of length of ten of interline space,  $l \sim 10^{-1}$  cm. This loop undergoes (on average) 10<sup>2</sup> reconnections per one second, or in other words it exists 10<sup>-2</sup>s without reconnection (as a whole). The own vortex filament dynamics (Kelvin waves dynamics) is much more slow process. For instances Kelvin wave signal runs around the loop in time of order  $l^2 / \kappa \approx 10$  seconds. Thus, the characteristic time of Kelvin waves dynamics exceeds time of existence of the loop by 10<sup>3</sup> times (!!!!).

# Random walking structure

- The structure of any loop is determined by numerous previous reconnections. Therefore any loop consists of small parts which "remember" previous collision. These parts are uncorrelated since deterministic Kelvin wave signals do not have a time to propagate far enough. Therefore loop has a structure of random walk (like polymer chain).

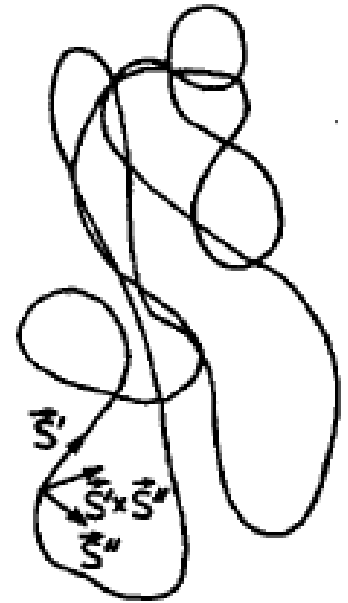


# Gaussian model of vortex loop

Main mathematical tool to describe random walk is the Wiener distribution. We use it in form, which allows to take into account possible anisotropy and finite curvature.

$$P(\{\mathbf{s}(\xi, t)\}) \propto \exp\left(-\int_0^l \int_0^l \mathbf{s}'^\alpha(\xi, t) \Lambda_{\alpha\beta}(\xi, \xi') \mathbf{s}'^\beta(\xi', t) d\xi d\xi'\right).$$

Here  $\Lambda_{\alpha\beta}(\xi, \xi')$  is Mexican-hat like function width  $\xi_0$ . The average loop can be imagine as consisting of many arches with mean radius of curvature equal  $\xi_0$  randomly (but smoothly) connected to each other. Quantity  $\xi_0$  is important parameter of the approach. It plays a role of the "elementary step" in the theory of polymer. It is low cut-of, theory does not describe scales smaller then  $\xi_0$ . Being a Gaussian function the Wiener distribution allows readily to calculate any average functional  $\langle A(\{\mathbf{s}(\xi, t)\}) \rangle$ .



# Statement of problem

The only degree of freedom of random walk is the length  $l$  of loop.

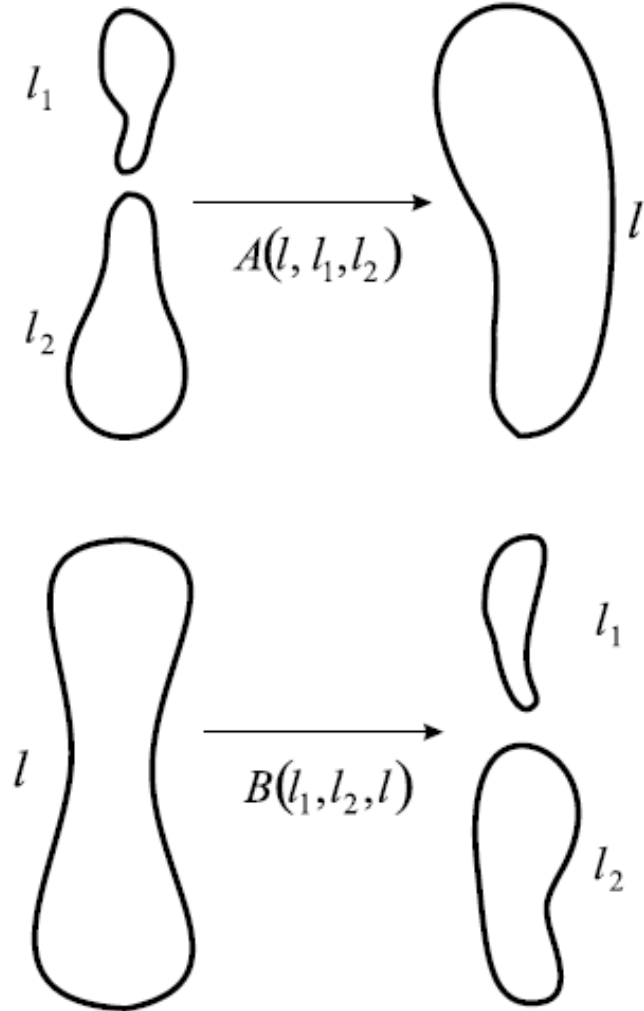
Let us introduce the distribution function  $n(l,t)$  of the density of a loop in the "space" of their lengths. It is defined as the number of loops (per unit volume) with lengths lying between  $l$  and  $l+dl$ . Knowing quantity  $n(l,t)$  and statistics of each personal loop we are able to evaluate various properties of real vortex tangle

# Evolution of $n(l,t)$

There are two main mechanisms for  $n(l,t)$  to be changed. The first one is related to deterministic motion (in fact to mutual friction shrinking or inflating loops).

The second mechanism is related to random processes of recombination. We take that splitting of loop into two smaller loops occurs with the rate of self-intersection (number of events per unit time)

$B(l_1, l_2, l)$ . The merging of loop occurs with the rate of collision  $A(l_1, l_2, l)$ .



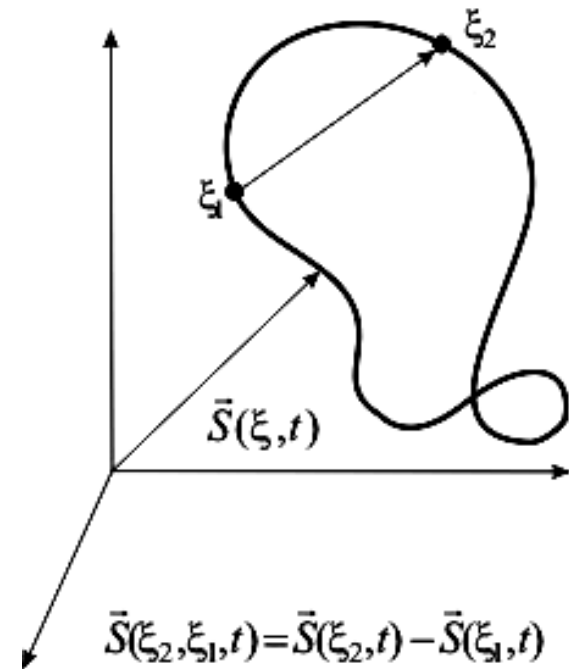
In view of what has been exposed above we can directly write out the master "kinetic" equation for rate of change the function  $n(l,t)$

$$\begin{aligned}
 & \frac{\partial n(l, t)}{\partial t} + \frac{\partial n(l, t)}{\partial l} \frac{\partial l}{\partial t} = \\
 & \int \int A(l_1, l_2, l) n(l_1) n(l_2) \delta(l - l_1 - l_2) dl_1 dl_2 \quad l_1 + l_2 \rightarrow l \\
 & - \int \int A(l_1, l, l_2, ) \delta(l_2 - l_1 - l) n(l) n(l_1) dl_1 dl_2 \quad l_1 + l \rightarrow l_2 \\
 & - \int \int A(l_2, l, l_1, ) \delta(l_1 - l_2 - l) n(l) n(l_1) dl_1 dl_2 \quad l_2 + l \rightarrow l_1 \\
 & - \int \int B(l_1, l_2, l) n(l) \delta(l - l_1 - l_2) dl_1 dl_2 \quad l \rightarrow l_1 + l_2 \\
 & + \int \int B(l, l_2, l_1) \delta(l_1 - l - l_2) n(l_1) dl_1 dl_2 \quad l_1 \rightarrow l + l_2 \\
 & + \int \int B(l, l_1, l_2) \delta(l_2 - l - l_1) n(l_1) dl_1 dl_2 \quad l_2 \rightarrow l + l_1
 \end{aligned}$$



# Evaluation of rates of self-intersection and collision

Let us take vector  $S$  connecting two points of the loop. Event  $S=0$  implies self-intersection of line with consequent reconnection and splitting of the loop. To find the rate of such events we have to find how often 3-component function  $S$  of 3 arguments vanishes. In other words we have to find number of zeroes of fluctuating function  $S$ .



# Coefficients $A(l_1, l_2, l)$ and $B(l_1, l_2, l)$

$$\sum \delta(\zeta - \zeta_a) = \left| \frac{\partial(X, Y, Z)}{\partial(\xi_2, \xi_1, t)} \right|_{\zeta=\zeta_a} \delta(\mathbf{S}_s(\xi_2, t, \xi_1, t_1))$$

Here  $X, Y, Z$  are the components of vector  $S(\xi_2, \xi_1, t)$ . If, further to introduces additional constraint  $\delta(\xi_2 - \xi_1 - l_1)$  and integrate over  $d\xi_1 d\xi_2$  we obtain the rate of self-crossing of line of full length  $l$  and breaking it down it into pieces  $l_1$  and  $l - l_1$ . In addition we have to do averaging over all possible fluctuating configurations.

$$B(l_1, l - l_1, l) = \int \int d\xi_1 d\xi_2 \delta(\xi_2 - \xi_1 - l_1) \left\langle \left| \frac{\partial(x, y, z)}{\partial(\xi_2, \xi_1, t)} \right|_{\zeta=\zeta_a} \delta(\mathbf{S}_s(\xi_2, t, \xi_1, t)) \right\rangle$$

$$A(l_1, l_2, l) = \frac{1}{V} \int \int d\mathbf{R}_1 d\mathbf{R}_2 \int \int d\xi_1 d\xi_2 \left\langle \left| \frac{\partial(x, y, z)}{\partial(\xi_2, \xi_1, t)} \right|_{\zeta=\zeta_a} \delta(\mathbf{S}(\xi_2, t, \xi_1, t)) \right\rangle$$

$$B(l_1, l_2, l) = \frac{1}{2} b_s V_l \frac{l}{(\xi_0 l_1)^{3/2}}, \quad A(l_1, l_2, l) = \frac{1}{2} b_m V_l l_1 l_2, \quad V_l \sim \kappa / \xi_0$$

By use of a special procedure (Zakharov ansatz) it can be shown that kinetic equation without "deterministic terms" has stationary power-like solution.

$$n(l) = C l^s$$

$$l = \tilde{l}_2 \left( l / \tilde{l}_2 \right), \quad l_1 = \tilde{l}_1 \left( l / \tilde{l}_2 \right), \quad l_2 = l \left( l / \tilde{l}_2 \right).$$

$$\begin{aligned} & \int \int A(l_1, l_2, l) n(l_1) n(l_2) \left( 1 - \left( \frac{l}{l_1} \right)^{4+2s} - \left( \frac{l}{l_2} \right)^{4+2s} \right) \delta(l - l_1 - l_2) dl_1 dl_2 \\ & - \int \int B(l_1, l_2, l) n(l) \left( 1 - \left( \frac{l}{l_1} \right)^{s+3/2} - \left( \frac{l}{l_2} \right)^{s+3/2} \right) \delta(l - l_1 - l_2) dl_1 dl_2. \end{aligned}$$

For  $s = -5/2$  both expressions in parentheses coincide with arguments of delta functions, thus the integrands include expressions of type  $(x) \delta(x)$  and these integrals vanish.

# Flux of length (energy) in space of the loops sizes

Distribution of loops over their lengths  $n(l) = C l^{-5/2}$  was frequently discussed early for various systems of one-dimensional singularities, however for thermodynamical equilibrium. Here we have a nonequilibrium state, with the flux of length (energy) in space of the loops sizes. The term "flux" here means just the redistribution of length among the loops due to reconnections.

$$\mathcal{L}(t) = \int l * n(l, t) dl$$

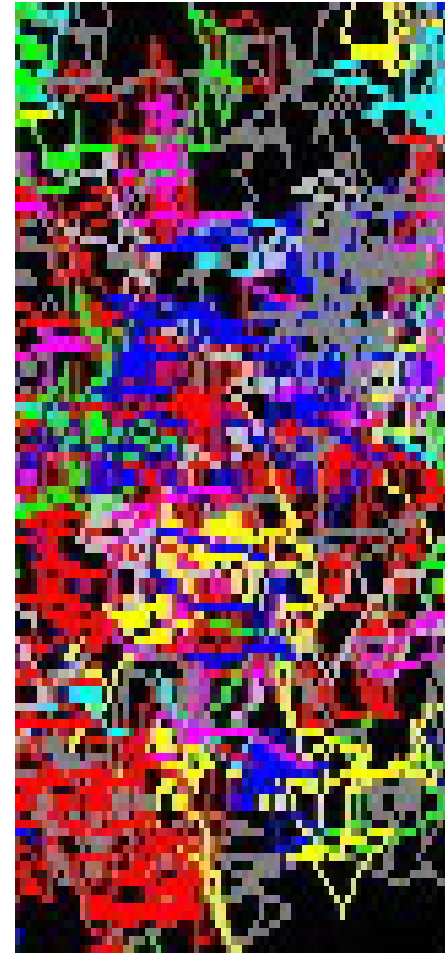
$$\frac{\partial \mathcal{L}(t)}{\partial t} + \frac{\partial P(l)}{\partial l} = 0.$$

$$P = 6.27C^2 b_m V_l - 2.77C b_s V_l \xi_0^{-3/2}.$$

# Low temperature case: Direct cascade

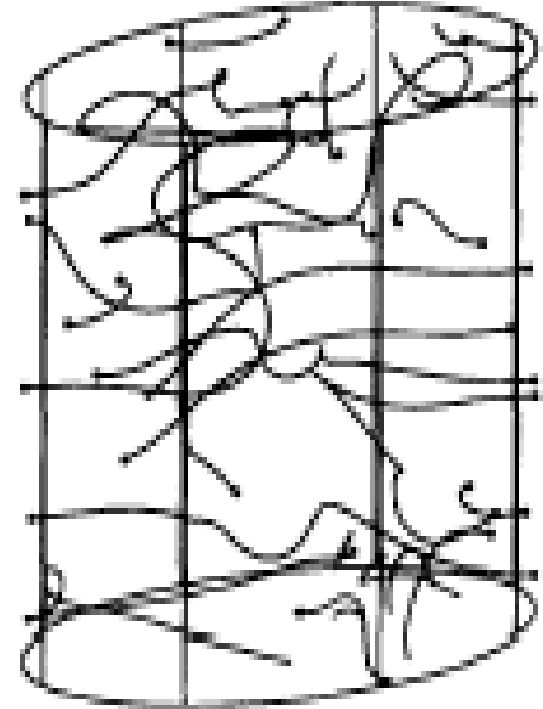
- Negative flux appears when break down of loops prevails and cascade-like process of generation of smaller and smaller loops forms. There exists a number of mechanisms of disappearance of rings on very small scales. It can be e.g. acoustic radiation, collapse of lines, Kelvin waves etc.

*Thus, in this case one can observe well-developed superfluid turbulence.*



# High temperature case: Inverse cascade

- The case with inverse is less clear. Inverse cascade implies the cascade-like process of generation of larger and larger loops. Unlike previous case of direct cascade, there is no apparent mechanism for disappearance of very large loops. The probable scenario is that parts of large loops are pinned on the walls. Finally a state with few lines stretching from wall to wall with poor dynamics and rare events is realized, this is a degenerated state of the vortex tangle.



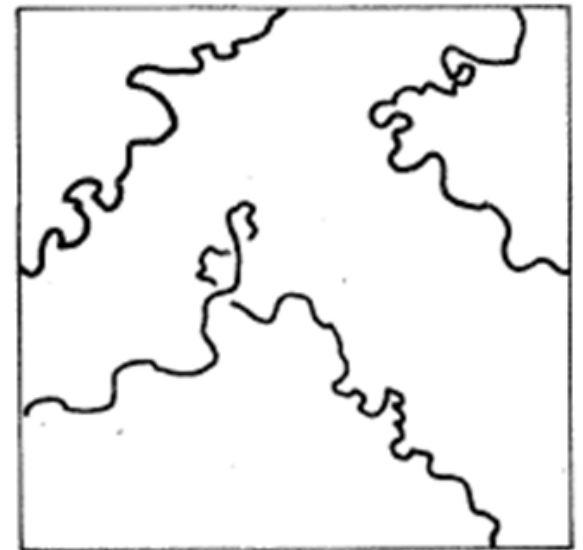
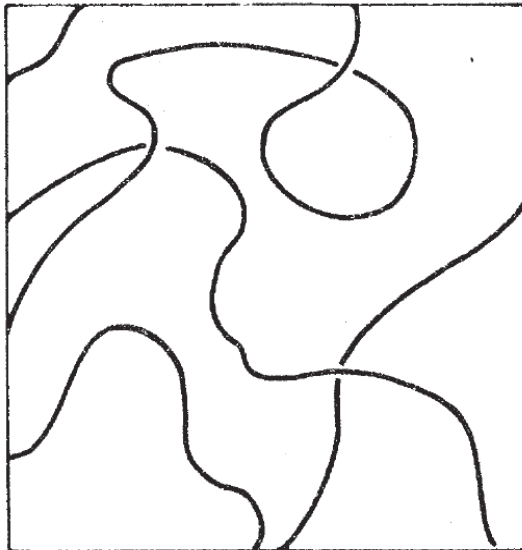
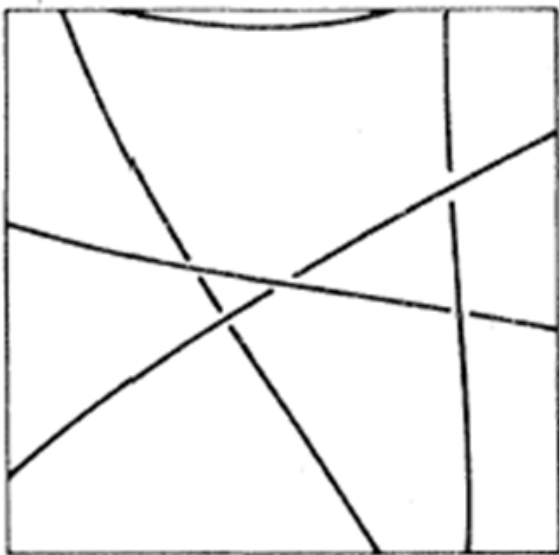
# Mean curvature and interline space

In steady case the positive flux of length exactly compensates the negative flux. This allows to find constant  $C$  and evaluate  $\mathcal{L}$

$$\mathcal{L} = \int_{\xi_0}^{\infty} l * n(l) dl = \frac{1}{2c_2^2 \xi_0^2}.$$

$$R \sim \delta = \mathcal{L}^{-1/2}$$

Result is very remarkable. The idea that interline space  $\delta = \mathcal{L}^{-1/2} \sim c_2 \xi_0$  is of order of mean radius of curvature was launched by Schwarz. Earlier it was confirmed only in numerical simulations. The theoretical and numerical values of  $c_2$  agree within 10%.





**The Rate of Reconnection.** The full rate of reconnection  $\dot{N}_{rec}$  can be evaluated directly from master kinetic equation. Indeed, this equation describes change of  $n(l)$  due to reconnection events. It takes into account sign of events, depending on whether the loop of size  $l$  appears or dies in result of reconnection. Therefore, if we take all terms in collision integral with the plus sign we obtain the total number of reconnections (in interval interval  $dl$ ).

$$\int \int A(l_1, l_2, l) n(l_1) n(l_2) \left( 1 + \left( \frac{l}{l_1} \right)^{4+2s} + \left( \frac{l}{l_2} \right)^{4+2s} \right) \delta(l - l_1 - l_2) dl_1 dl_2$$

$$+ \int \int \int B(l_1, l_2, l) n(l) \left( 1 + \left( \frac{l}{l_1} \right)^{s+3/2} + \left( \frac{l}{l_2} \right)^{s+3/2} \right) \delta(l - l_1 - l_2) dl_1 dl_2$$

Evaluating  $\dot{N}_{rec}$  for  $s = -5/2$  we obtain

$$\dot{N}_{rec} = \frac{1}{3} \frac{\kappa(b_s C_{VLD} + b_m^2 C_{VLD})}{\xi_0^5} = C_{rec} \kappa \mathcal{L}^{5/2}$$

Where  $C_{rec}$  one more constant running in interval 0.1 – 0.5. That results also agrees with the recent numerical investigation by Barenghi and Samuels.

# Conclusion (to Part II)

- We demonstrated that the dynamics of vortex tangle is satisfactorily described in language of the kinetics of reconnecting (splitting and merging) Brownian loops.
- The intrinsic (deterministic) dynamics of vortex lines is secondary in value.
- Thus, the optimistic view is such that by varying the characteristics of the Brownian loop, one can describe the variety of phenomena of quantum turbulence. The pessimistic point of view is that this is impossible and the full solution of the problem requires something like string field theory for nonlinear strings.
- A pessimist is a well informed optimist.

**Thank You !**