Laboratory of Topology and Dynamics,
Novosibirsk State University

Conference on Physical Knotting, Vortices and Surgery in Nature*

June 3 — August 27, 2020

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Conference Schedule

June 3 at 10:00 PM by Chicago time (GMT-5) **Louis Kauffman**, *Knot Cobordism and Vortex Reconnection*

June 10 at 10:00 AM by Bucharest time (GMT+3) **Marius Buliga**, *Emergent rewrites in knot theory and logic*

June 17 at 10:00 AM by Milan time (GMT+2) **Renzo Ricca**, *Topological cascade through vortex reconnection*

June 24 at 10:00 AM by Tallahassee time (GMT-4) **De Witt Sumners**, *Reconnection in Biology and Physics*

July 1 at 12:30 by Bangalore time (GMT+5:30) **Rukhsan Ul Haq**, *Topological Protection of quantum states for quantum computation*

July 8 at 10:00 AM by Chattanooga time (EST) **Eleni Panagiotou**, *Entanglement of open curves*

July 15 at 16:30 by Istanbul time (GMT+3) **Neslihan Gugumcu**, *What is a braidoid diagram?*

July 22 at 16:00 by Moscow time (GMT+3) **Vassily Manturov**, *3-free links, braids, G_n^k groups and link-homotopy*

July 29 at 21:00 by Novosibirsk time (GMT+7) **Sergey Nemirovskii**, *Recombination of Vortex Loops in HeII and Theory of Quantum Turbulence*

August 5 at 21:00 by Novosibirsk time (GMT+7) **Timur Nasybullov**, *Virtual quandle for links in lens spaces*

August 12 at 21:00 by Novosibirsk time (GMT+7) **Sergey Alekseenko, Pavel Kuibin**, *Vortex Reconnection in a Swirl Flow*

August 19 at 17:00 by Mohali time (GMT+5:30) **Manpreet Singh**, *Virtually symmetric representations and marked Gauss diagrams*

August 26 at 10:00 PM by Chicago time (GMT-5) **Louis Kauffman**, *The Dirac Equation and the Majorana Dirac Equation*

KNOT COBORDISM AND VORTEX RECONNECTION

LOUIS H. KAUFFMAN

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This talk (joint work with William Irivine) will discuss the use of properties of cobordism and concordance of classical knots to give lower bounds on the reconnection number of knotted vortices.

EMERGENT REWRITES IN KNOT THEORY AND LOGIC

MARIUS BULIGA

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I explain in what sense new graph rewrite systems emerge from given ones, with two examples:
(1) the emergence of the $\mathbb{R}^3$ (Reidemeister 3) rewrite from $\mathbb{R}^1, \mathbb{R}^2$ and some uniform continuity assumptions, and relations to curvature,
(2) the emergence of the beta rewrite in lambda calculus from the shuffle rewrite and relations to the commutativity of the addition of vectors in the tangent space of a manifold.

TOPOLOGICAL CASCADE THROUGH VORTEX RECONNECTION

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Complex tangles of vortex filaments are ubiquitous in classical and quantum turbulent flows. In recent years remarkable progress has been made to produce vortex knotting in laboratory and investigate the topological cascade driven by a stepwise unlinking process through single reconnection events. During the process helicity, a fundamental invariant of ideal fluid mechanics, may change due to the change in linking, writhe and twist of interacting structures. Here we show how adapted knot polynomials such as HOMFLYPT (or Jones), derived from helicity, can be used to detect topological cascade and provide some quantitative information on topological states [1], [2]. Since vortex reconnection is responsible for topological transition we then focus on the fundamental mechanism that governs vortex reconnection. Since during reconnection writhe remains conserved [3], we highlight the role of twist responsible to energy transfer and depletion.

References

[1] X. Liu, R.L. Ricca, On the derivation of HOMFLYPT polynomial invariant for fluid knots. J. Fluid Mech. (2015), 773, 34–48.
[2] X. Liu, R.L. Ricca, Knots cascade detected by a monotonically decreasing sequence of values. Nature Sci. Rep. 6(2016), 24118.
[3] C.E. Laing, R.L. Ricca, DeW.L. Sumners, Conservation of writhe helicity under anti-parallel reconnection. Nature Sci. Rep. 5(2015) 9224.

RECONNECTION IN BIOLOGY AND PHYSICS

DE WITT SUMNERS

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Reconnection is a fundamental event in many areas of science, including the interaction of vortices in classical and quantum fluids, magnetic flux tubes in magnetohydrodynamics and plasma physics, and site-specific recombination in DNA. The helicity of a collection of flux tubes can be calculated in terms of writhe, twist and linking among tubes. We prove that the writhe helicity is conserved under anti-parallel reconnection [1]. We will discuss the mathematical similarities between reconnection events in biology and physics, and the relationship between iterated reconnection and curve topology. We will discuss helicity and reconnection in a tangle of confined vortex circles in a superfluid.

References

[1] C.E. Laing, R.L. Ricca, DeW.L. Sumners, Conservation of writhe helicity under anti-parallel reconnection. Nature Sci. Rep. 5(2015), 9224.

TOPOLOGICAL PROTECTION OF QUANTUM STATES FOR QUANTUM COMPUTATION

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Quantum states are known to be fragile which poses challenges for quantum computation in which degenerate states are used for storing the quantum information. The fragility of the quantum states also makes them very prone to the environmental perturbations and noises. It turns out that topology offers us a way out for this problem. In our talk, we will highlight some aspects of how the interplay between topology and quantum physics offers us the ways to make quantum information more protected. We will take an algebraic route to demonstrate the resilience of the topological quantum phases against the environmental perturbations.

ENTANGLEMENT OF OPEN CURVES

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Open curves in space can entangle and even tie knots, a situation that arises in many physical systems of filaments. To measure entanglement of open curves it is natural to look for measures of complexity in the study of knots and links. In this talk we will see how the Gauss linking integral can be applied to open curves and also show that the information it captures is useful in our understanding of polymer mechanics and dynamics. In this talk we will also seek stronger measures of entanglement of open curves and provide a framework within which knot and link polynomials can be rigorously defined for open curves in 3-space. In particular, we will define the Jones polynomial of open curves in 3-space and discuss some of its properties.

WHAT IS A BRAIDOID DIAGRAM?

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In this talk we first review the basics of the theory of knotoids introduced by Vladimir Turaev in 2012 [1]. A knotoid diagram is basically an open-ended knot diagram with two open endpoints that can lie in any local region complementary to the plane of the diagram. The theory of knotoids extends the classical knot theory and brings up some interesting problems and features such as the height problem [1],[3] and parity notion and related invariants such as off writhe and parity bracket polynomial [4]. It was a curious problem to determine a "braid like object" corresponding to knotoid diagrams. The second part of this talk is devoted to the theory of braidoids, introduced by the author and Sofia Lambropoulou [2]. We present the notion of a braidoid and analogous theorems to the classical Alexander Theorem and the Markov Theorem, that relate knotoids/multi-knotoids in the plane to braidoids.

References

[1] V. Turaev, Knotoids, Osaka J. of Mathematics 49(2012), 195-223.
[2] N. Gugumcu, S. Lambropoulou, to appear in Israel J. of Mathematics
[3] N. Gugumcu, L. Kauffman, New Invariants of Knotoids, European J. of Combinatorics, (2017), 65C, 186–229
[4] The Guassian parity and minimal diagrams of knot-type knotoids, submitted.

3-FREE LINKS, BRAIDS, G_N^K GROUPS AND LINK-HOMOTOPY

VASSILY MANTUROV

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In this talk, we combinatorially define free 3-links and construct well defined mapping from oriented classical links in $\mathbb{R}^3$ to free 3-links (with end points). And then, we will talk about possible modifications and invariants of free 3-links, which are related with free knots and invariant valued in pictures.
This work is joint with D.A. Fedoseev and S. Kim.

RECOMBINATION OF VORTEX LOOPS IN HEII AND THEORY OF QUANTUM TURBULENCE

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The term "quantum turbulence" (QT) unifies the wide class of phenomena where the chaotic set of one-dimensional quantized vortex filaments (vortex tangles) appear in quantum fluids and greatly influence various physical features. Quantum turbulence displays itself differently depending on the physical situation, and ranges from quasi-classical turbulence in flowing fluids to a near-equilibrium set of loops in phase transition. The statistical configurations of the vortex tangles are certainly different in, say, the cases of counterflowing helium and a rotating bulk, but in all the physical situations very similar theoretical and numerical problems arise. Furthermore, quite similar situations appear in other fields of physics, where a chaotic set of one-dimensional topological defects, such as cosmic strings, or linear defects in solids, or lines of darkness in nonlinear light fields, appear in the system. There is an interpenetration of ideas and methods between these scientific topics which are far apart in other respects. The first part of the report is introductory, it presents a short overview of the work on quantum turbulence. History questions, main trends and key results are exposed. In the second part, a theory is developed to describe the superfluid turbulence on the base of kinetics of the merging and splitting vortex loops. Because of very frequent reconnections the vortex loops as a whole do not live long enough to perform any essential evolution due to the deterministic motion. On the contrary, they rapidly merge and split and these random recombination processes prevail over other slower dynamic processes. To develop quantitative description we take the vortex loops to have a Brownian structure with the only degree of freedom, which is the length l of the loop. We perform investigation on the base of the Boltzmann type kinetic equation for the distribution function nl of number of loops with length l. Analyzing the solution of this kinetic equation we drew several results on the structure and dynamics of the vortex tangle in the turbulent superfluid helium.

VIRTUAL QUANDLE FOR LINKS IN LENS SPACES

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Over the years knot theory has worked with knots and links in the three-dimensional sphere S^3. However, together with improving knowledge about 3-manifolds, great attention has been paid to knots and links in manifolds different from S^3. The most studied (afterS^3) manifolds where knots and links were considered are lens spaces S^3. There are interesting articles explaining applications of knots in lens spaces to other fields of science: [1] exploits them to describe topological string theories and [2] uses them to describe the resolution of a biological DNA recombination problem. A lot of link invariants can be generalized from links in S^3 to links in $L(p, q)$. Kauffman bracket skein module, knot Floer homology, HOMFLY-PT polynomial all have analogues in lens spaces. Despite the fact that some invariants can be generalized to links in lens spaces, sometimes it is very difficult to use them. For example, the fundamental quandle of a link which has a very simple topological description for links in S^3 can be easily generalized to links in $L(p, q)$. However, an explicit procedure to write down a presentation of the fundamental quandle for links in $L(p, q)$ by generators and relations directly from a diagram is known only in the case $(p, q) = (2, 1)$ (i. e. in the case of the projective space), so it is almost impossible to use this invariant. Another disadvantage of the ordinary fundamental quandle for links in $L(p, q)$ is that the fundamental quandle of the link K in $L(p, q)$ is isomorphic to the fundamental quandle of its lift in S^3. So it cannot distinguish different links with equivalent liftings. During the talk we are going to introduce the so called virtual quandle for links in lens spaces $L(p, q)$, with $q = 1$. This invariant has two valuable advantages over an ordinary fundamental quandle for links in lens spaces: the virtual quandle can distinguish links with equivalent liftings and the presentation of the virtual quandle can be easily written from the band diagram of a link. Virtual quandles were fintroduced by V. Manturov as generalizations of quandles for virtual links. During the talk we are not going to work with virtual links, however, we will use the term "virtual quandle" since it is a common term.

References

[1] S. Stevan, Torus knots in lens spaces and topological strings, Ann. Henri Poincare, V. 16, 8(2015), 1937–1967.

[2] D. Buck, M. Mauricio, Connect sum of lens spaces surgeries: application to Hin recombination, Math. Proc. Cambridge Philos. Soc., V. 150, 2011, 505–525.

VORTEX RECONNECTION IN A SWIRL FLOW

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Vortex reconnection seems to be a fundamentally important phenomenon resulting in a drastic change in the topology of vortex structures. This paper presents an overview of literature data on vortex reconnection and our results of experimental study of vortex interactions and processes of vortex reconnection on helical vortices in a swirl flow. The experimental setup is a simplified model of a draft tube of hydro turbine. The result of reconnection can be either formation of an isolated vortex ring while preserving the basic spiral vortex tube or formation of a system consisting of the vortex ring linked with the spiral tube. On the original spiral in the reconnection zone, the left-handed Kelvin wave, running up the vortex tube, is generated consistently. A number of topological features of vortex reconnection, such as asymmetry of the process near the ring and spiral tube, formation of two bridges and two threads, as well as formation of external bridges, not associated with the vortex reconnection process, were revealed. Theoretically and experimentally, an explanation is given for the random shocks that occur in real water turbines. This phenomenon is due to the interaction between the solid wall and the vortex ring formed as a result of reconnection. The question of how exactly reconnections affect the formation of an energy cascade in a turbulent flow is discussed. The results obtained are useful for understanding and describing processes in the vortex chambers and draft tube of a hydro turbine, quantum and conventional turbulence, and solar flares.

VIRTUALLY SYMMETRIC REPRESENTATIONS AND MARKED GAUSS DIAGRAMS

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In this talk, we will define virtually symmetric representations of virtual braid group VB_n and show that many previously known representations are equivalent to virtually symmetric representations. Using a virtually symmetric representation, we will associate groups to virtual links and study group system of virtual knots by defining marked Gauss diagrams as an extension of Gauss diagrams. In particular, we will extend the definition of virtual link group to marked Gauss diagrams and define peripheral structure for 1-circle marked Gauss diagrams. We will define C_m-groups and prove that every irreducible C_1-group can be realized as the group of a marked Gauss diagram. We will give an interpretation of marked Gauss diagrams in terms of virtual spatial graph diagrams with marked nodes. Also, we will extend many results proved by S. G. Kim in [1] to marked Gauss diagrams.
This is a joint work with Valeriy Bardakov and Mikhail Neshchadim.

References

[1] S.G. Kim, Virtual knot groups and their peripheral structure, J. Knot Theory Ramifications, 9(2000), no. 6, 797–812.

THE DIRAC EQUATION AND THE MAJORANA DIRAC EQUATION

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The Dirac Equation was discovered by P.A.M Dirac by using the relation among energy (E), momentum (p) and rest mass (m) given by special relativity (with c = speed of light = 1 in this formulation): $E^2 = p^2 + m^2$. Dirac observed that if a and b generate a Clifford algebra so that $a^2 = b^2 = 1$ and $ab + ba = 0$, then we can write $E = ap + bm$, and it will follow formally that $E^2 = p^2 + m^2$. This allowed an effective way to obtain a square root for the quantum mechanical energy operator, and the possibility to write an analog of the Schrodinger equation that is relativistically invariant. The resulting equation is called the Dirac equation. Ettore Majorana in the 1930s investigated a version of the Dirac equation that (by careful choice of the Clifford algebra) has real solutions, and he suggested that these solutions corresponded to particles that are their own anti-particles. Majoranas ideas have continued into the present day, and are now being applied in condensed matter physics. Majorana Fermions are believed to exist in these contexts and the mathematics suggests that they are related to certain representations of the Artin braid group, hence of possible use in topological quantum computing. In this talk, we revisit the Clifford algebra structure associated with Majorana Fermions and the Majorana Dirac equation. We show how the nilpotent algebraic method of Rowlands leads to actual real solutions to the Majorana Dirac equation, and we relate these solutions to the Feynman checkerboard model in the case of one dimension of space and one dimension of time. Time permitting we will discuss the possible topological meaning of this approach. This talk is joint work with Peter Rowlands.

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