Non-homogeneous Bracket

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The oriented bracket

The relations among coefficients

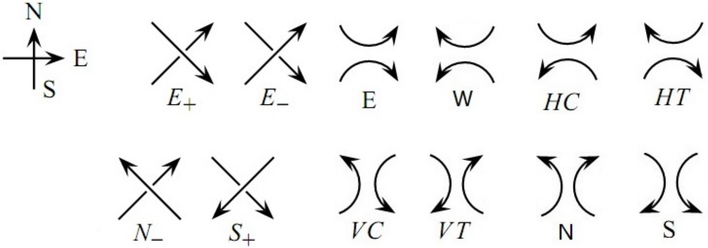
Non-homogeneous invariant

A related invariant

Part I . The oriented bracket

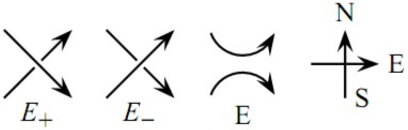
Some notations

We use the following symbols to denote crossings of link diagrams:



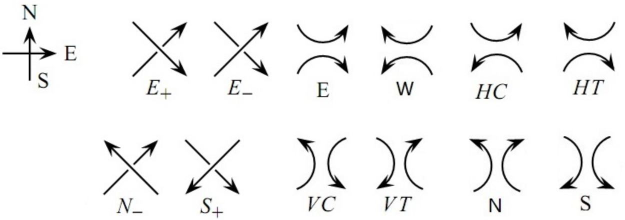
Some Classical knot polynomials

Conway polynomial $\nabla_{E_+}(z) - \nabla_{E_-}(z) = z\nabla_E(z)$
Jones polynomial $t^{-1}J_{E_+}(t) - tJ_{E_-}(t) = (t^{1/2} - t^{-1/2})J_E(t)$
HOMFLYPT polynomial. $\frac{1}{v}P_{E_+}(v,z) - vP_{E_-}(v,z) = zP_E(v,z)$
Kauffman polynomial and some others.



Those equations are called skein relations. They are homogeneous equations.

We propose the following skein equations:



- When the two strands of $E_\pm$ are from same component, they satisfies the following relation:
  $f(E_+) = c_1 f(E) + c_2 f(HC) + df(VC)$
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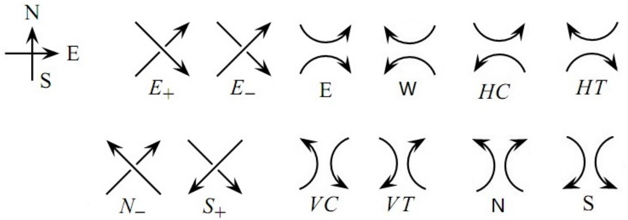

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- If they are from different components:
  $f(E_+) = c' f(E) + d' f(S) + \beta$
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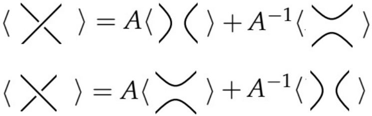
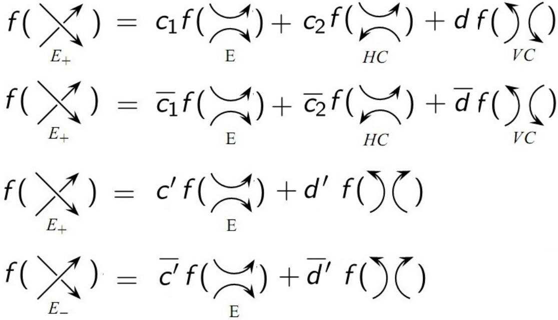
Compare to the Kauffman Bracket

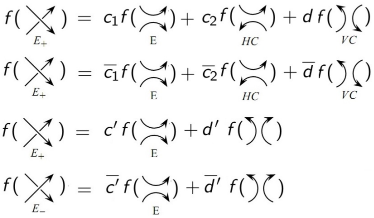
- The Kauffman Bracket



- The oriented Bracket without constant terms.



On Orientation

- 

- Lemma 1: Let K be an oriented, $-K$ denotes K with orientation reversed. Then $f(K) = f(-K)$.

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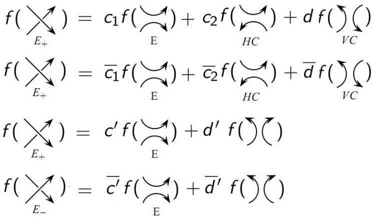
- 

- Lemma 1: Let K be an oriented, $-K$ denotes K with orientation reversed. Then $f(K) = f(-K)$.
- Lemma 2: Let L be an oriented 2-component link, $-L$ denotes L with all orientation reversed. Then $f(L) = f(-L)$.

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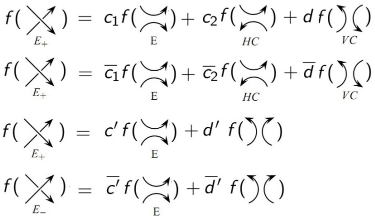
- 

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- Lemma 2: Let L be an oriented 2-component link, $-L$ denotes L with all orientation reversed. Then $f(L) = f(-L)$.
- A more general version will contain more terms, like $f(W), f(HT), f(VT), \cdots$. By the above lemmas, they are equivalent to the above invariant.

A subtlety

- When the two strands of $E_\pm$ are from same component, they satisfies the following relation:
  $f(E_+) = c_1 f(E) + c_2 f(HC) + d f(VC)$
  $f(E_-) = \bar{c}_1 f(E) + \bar{c}_2 f(HC) + \bar{d} f(VC) + \alpha$
- If they are from different components:
  $f(E_+) = c' f(E) + d' f(S) + \beta$
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- $f(O \cup K) = v f(K)$

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- **Rule**: If there are crossings involves different components, then first apply skein relations to those crossings.

Calculation of the invariant :

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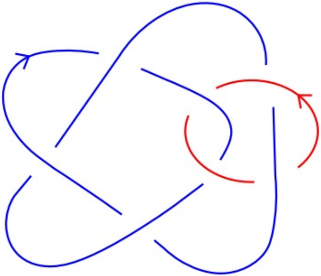
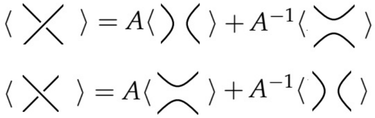


$7a3$

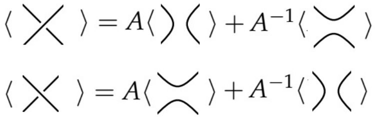
Whenever we have two non-split components, we deal with their intersection points first.

Compare to the Kauffman Bracket

Definition of Kauffman Bracket (L. Kauffman, 1987):
For an unoriented diagram D, $\langle D \rangle$ is a Laurent polynomial in a single variable A defined by the three following axioms.
1. $\langle \bigcirc \rangle = 1$ where $\bigcirc$ denotes the diagram of unknot with no crossings.
2. Delta: $\langle D \cup \bigcirc \rangle = \delta \langle D \rangle$ where $\delta = -A^{-2} - A^{2}$.
$\langle D \cup \bigcirc \rangle$ denotes the diagram D together with a single component that does not cross itself or D.
3. Skein relation:



Kauffman Bracket



Calculation: The bracket polynomial can be calculated in two ways.

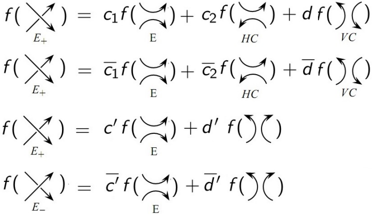
1. Inductively use the skein relation.
2. Simultaneously apply the skein relation to all crossings.

$\langle L \rangle = \sum_S A^{a(s)} A^{-b(S)} (-A^2 - A^{-2})^{|S|-1}$

This is usually called the state sum formula. It is easier to use and prove properties of the Kauffman Bracket. Hence this is very useful.

On the state sum formula of the invariant:

Rule: If there are crossings involves different components, then first apply skein relations to those crossings.



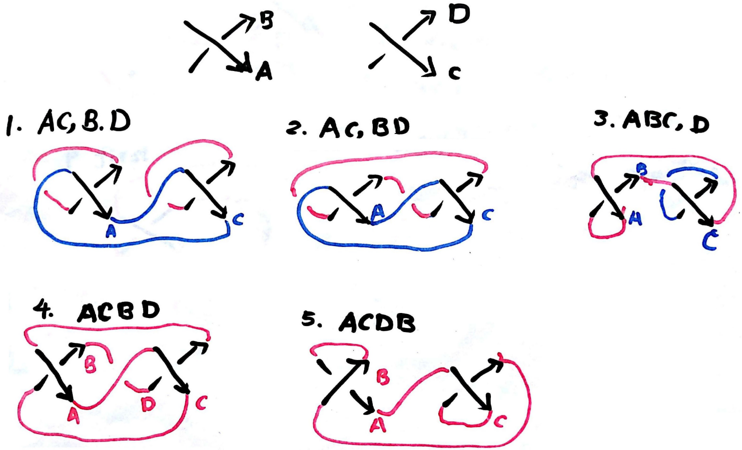
This invariant also has a state sum formula.

The final result looks like a summation of states, but the coefficients depends on the special order of crossings to apply the skein relations.

Part II . The relations among coefficients

Equations for two crossings

$F_{pq} = F_{qp}$

- 

- **We only need equation sets 2,4.**

Equations for two crossings

$F_{pq} = F_{qp}$
Case 2, (AC, BD) Resolving p first, we shall get the following equations.

- $(E_+, E_+) = (c'E, E_+) + (d'S, S_-)$

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- $(E_+, E_+) = (c'E, E_+) + (d'S, S_-)$
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- $(E_+, E_+) = (c'E, E_+) + (d'S, S_-) =$
  $c'c_1(E, E) + c'c_2(HC, HT) + d'\overline{d}(VC, HC) + \cdots$

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- Likewise, if we resolving q first, we shall get
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- Compare the two results we get: $d'\overline{d} = c_1'd, \cdots$
- We switch the order of the second term $d'\overline{d} = dc_1'$. The the first variable of both sides are from crossing point p.

Completing the relations

$F_{pq} = F_{qp}$

- If we have an equation like $xy = zw$, then we write $\overline{x}y = \overline{z}w$, $x\overline{y} = z\overline{w}$, $\overline{xy} = \overline{zw}$. This is called completing the relations.

Completing the relations

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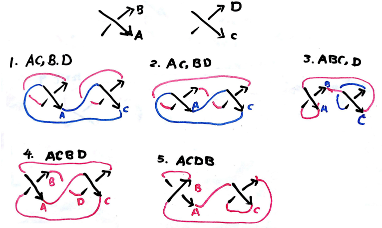
- 

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- Those relations are followed from case 2 by changing the crossing from $=$ to $-$.
- For example, from $d'\overline{d} = dc_1'$ we get $\overline{d}'\overline{d} = \overline{d}c_1'$, $d'd = d\overline{c_1}'$, $\overline{d}'d = \overline{dc}_1'$

Coefficients relations from $F_{pq} = F_{qp}$:

$d'\overline{d} = dc'$, $\overline{d}'\overline{d} = \overline{d}c'$, $d'd = d\overline{c}'$, $\overline{d}'d = \overline{d}\overline{c}'$,
$d\overline{d} = \overline{c}'c_2 + d'c_1$, $\overline{dd} = c'c_2 + \overline{d}'c_1$, $dd = \overline{c}'\overline{c_2} + d'\overline{c_1}$,
$\overline{d}d = c'\overline{c_2} + \overline{d}'\overline{c_1}$
$\overline{d}'\overline{c_1} = c_1d'$, $\overline{d}'\overline{c_2} = c_2d'$, $\overline{d}\overline{c_2} = c_1d$, $\overline{d}\overline{c_1} = c_2d$,
$d\overline{c_2} = \overline{c_1}d$, $\overline{c'}c_1 = \overline{c_1}c'$, $\overline{c'}c_2 = \overline{c_2}c'$,

Redermeister move invariance :

Ω_1: $1 - d = (c_1 + c_2)v, \quad 1 - \overline{d} = (\overline{c}_1 + \overline{c}_2)v,$

Ω_2: $v = c' + d' \;, \quad v = \overline{c}' + \overline{d}',$

$f(O \cup K) = \color{red}{v}\color{black} f(K)$

Redermeister move I,II invariance $\Rightarrow$ Redermeister move three invariance

Then $f(D)$ is a knot invariant.

Regular invariant :

Ω_2: $\quad v = (c'(\overline{c}_1 + \overline{c}_2) + d'(c_1 + c_2))v + (c'\overline{d} + dd')$,

$1 = (c_1 v + c_2 v + d)(\overline{c}_1 v + \overline{c}_2 v + \overline{d})$

$f(O \cup K) = v f(K)$

$f(s^+) = (c_1 v + c_2 v + d) f(K)$, $f(s^-) = (\overline{c}_1 v + \overline{c}_2 v + \overline{d}) f(K)$

If diagram D has w_+ positive crossings, w_- negative crossings, then

$F(D) = (c_1 v + c_2 v + d)^{-w_+} (\overline{c}_1 v + \overline{c}_2 v + \overline{d})^{-w_-} f(K)$ is a knot invariant.

Calculation 2. Word problem and Normal form

Although the result is unique modulo the relations, but it is hard to compare two results.
Different ways of calculation may give different results.
However, using the Diamond lemma, one can get a unique normal form for the result.

Calculation 2. Word problem and Normal form

$d'd' = c'c'$, $d'd = bdc'$, $\cdots$
Can be regarded as rewriting rules.
$d'd' \to c'c'$, $d'd \to bdc'$, $\cdots$

An Equivalent version of the diamond lemma.

For every binary relation with no decreasing infinite chains and satisfying the diamond property, there is a unique minimal element in every connected component of the relation considered as a graph.

$$\begin{tikzcd}
 & w \arrow[dl] \arrow[dr] & \\
u \arrow[dr] & & v \arrow[dl] \\
 & t &
\end{tikzcd}$$

Figure: The diamond property.

Remark 1

Given any oriented link diagram, one can calculate $f(D)$, then one can get a unique normal form of $f(D)$.

Part III . Non-homogeneous invariant

Homogeneous and non-homogeneous equations

Homogeneous equation: $ax + by + cz = 0$
Non-homogeneous equation $ax + by + cz = d$
Here a, b, c, d are constants, x, y, z are variables.

Classical knot polynomials are homogeneous

Conway polynomial $\nabla_{E_+}(z) - \nabla_{E_-}(z) = z\nabla_E(z)$
Jones polynomial $t^{-1}J_{E_+}(t) - tJ_{E_-}(t) = (t^{1/2} - t^{-1/2})J_E(t)$
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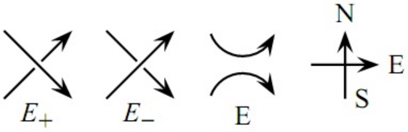

First approach to non-homogeneous invariant fails

$$af_{E_+}(t) + bf_{E_-}(t) + cf_E(t) = 0$$
$$ag_{E_+}(t) + bg_{E_-}(t) + cg_E(t) = \beta$$

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$$ag_{E_+}(t) + bg_{E_-}(t) + cg_E(t) = \beta$$

- If we change variable and let $\beta = (a+b+c)\gamma$, then $ag_{E_+}(t) + bg_{E_-}(t) + cg_E(t) = (a+b+c)\gamma$

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- If we change variable and let $\beta = (a+b+c)\gamma$, then $ag_{E_+}(t) + bg_{E_-}(t) + cg_E(t) = (a+b+c)\gamma$
- $a(g_{E_+}(t) - \gamma) + b(g_{E_-}(t) - \gamma) + c(g_E(t) - \gamma) = 0$

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- If we change variable and let $\beta = (a+b+c)\gamma$, then $a g_{E_+}(t) + b g_{E_-}(t) + c g_E(t) = (a+b+c)\gamma$
- $a(g_{E_+}(t) - \gamma) + b(g_{E_-}(t) - \gamma) + c(g_E(t) - \gamma) = 0$
- Hence $f(t) = g(t) - \gamma$

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- $a(g_{E_+}(t) - \gamma) + b(g_{E_-}(t) - \gamma) + c(g_E(t) - \gamma) = 0$
- Hence $f(t) = g(t) - \gamma$
- This gives a trivial non-homogeneous invariant.

We need system of equations

$$
\begin{aligned}
&af_1(t) + bf_2(t) + cf_3(t) + df_4(t) = \beta \\
&ef_1(t) + ff_2(t) + gf_3(t) + hf_4(t) = 0
\end{aligned}
$$

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- $a(f_1(t) - \gamma) + b(f_2(t) - \gamma) + c(f_3(t) - \gamma) + d(f_4(t) - \gamma) = 0$ $e(f_1(t) - \gamma) + f(f_2(t) - \gamma) + g(f_3(t) - \gamma) + h(f_4(t) - \gamma) = -(e+f+g+h)\gamma$

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- If we change variable and let $\beta = (a+b+c+d)\gamma$, then $af_1(t) + bf_2(t) + cf_3(t) + df_4(t) = (a+b+c+d)\gamma$
- $a(f_1(t) - \gamma) + b(f_2(t) - \gamma) + c(f_3(t) - \gamma) + d(f_4(t) - \gamma) = 0$
  $e(f_1(t) - \gamma) + f(f_2(t) - \gamma) + g(f_3(t) - \gamma) + h(f_4(t) - \gamma) = -(e+f+g+h)\gamma$
- Hence $g(t) = f(t) - \gamma$ is still a non-homogeneous invariant.

We propose the following skein equations:

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Non-homogeneous term:
We need the following to get a knot invariant.

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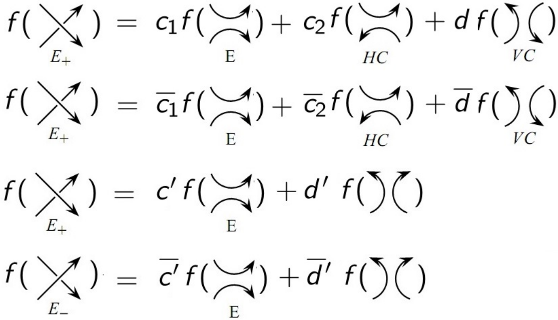
- In the future, we need to calculate the complete rewriting rule and get some concrete example.

Part IV . A related invariant

A related invariant

- The oriented Kauffman Bracket without constant terms.



- $E_+ + bE_- = (c_1 + b\overline{c}_1)E + (c_2 + b\overline{c}_2)HC + (d + b\overline{d})VC,$
  $E_+ + b'E_- = (c' + b'\overline{c}')E + (d' + b'\overline{d}')S,$

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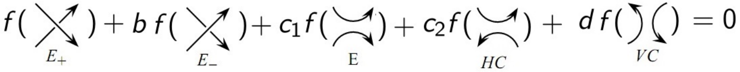
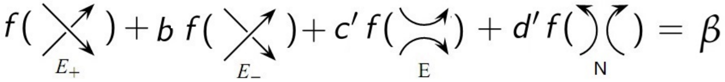
- This invariant is related to a more general invariant.
- 

- 

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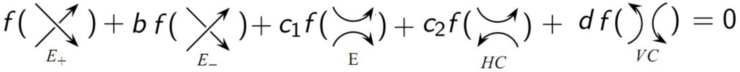
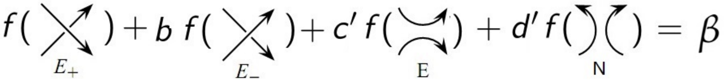
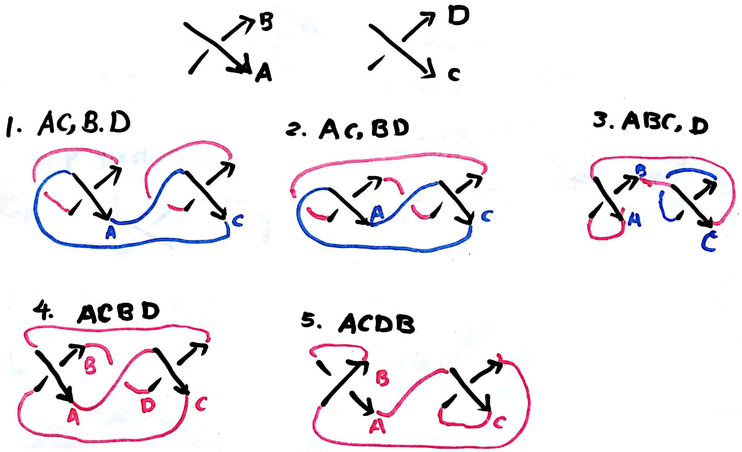
  $E_+ + b'E_- = (c' + b'\overline{c}')E + (d' + b'\overline{d}')S$,
- This invariant is related to a more general invariant.
- When the two strands of $E_\pm$ are from same component, they satisfies the following relation:
- 

- If they are from different components:
- 

We propose the following skein equations:

- 

- We guess that we do not need all equation sets 1-5. But we have not finished the proof yet.

Relation with other polynomials

Compare to the well-known knot polynomials.

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- (3) If we set $c_1 = c_2 = c_3 = c_4 = -z/4, c_1' = c_2' = -z/2.d_1 = d_2 = d_1' = d_2' = z/2$, and $b = b' = -1$, and modify it by writhe, then we can get the 2-variable Kauffman polynomial.

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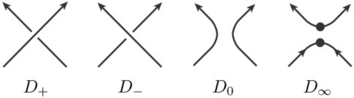
- (4) Hence it is a generalization of both HOMFLY and 2-variable Kauffman polynomial.

Related works

- (1) New invariants of links and their state sum models, Louis H. Kauffman, Sofia Lambropoulou, arXiv:1703.03655.
  (2) Their invariant is the usual HOMFLY polynomial when it is restrict to knots.
  (3) When restrict to knots, our invariant usually contains all the variables.

YASUYUKI MIYAZAWA's approach

- (1) A link invariant dominating the HOMFLY and the Kauffman polynomials, Journal of Knot Theory and Its Ramifications, November 2010, Vol. 19, No. 11 : pp. 1507-1533, YASUYUKI MIYAZAWA.
- $H_{D_+} + abH_{D_-} + zH_{D_0} + wH_{D_\infty} = 0$,



(2) Their way is a "local resolution", ours is a "global resolution".

Last page

Thank you for attention!